Answers to Selected Exercises

Chapter 1

Lesson 1.1

11. a. 120 720

b. The number of schedules possible when there are n choices is n times the number of schedules possible when there are $n - 1$ choices.

12. 8 12 17

Lesson 1.2

5. a.

A	30.8%	69.2%
B	19.2%	0.0%
C	23.1%	0.0%
D	26.9%	30.8%

7. Plurality: B Borda: D Runoff: C Sequential runoff: C

11. $C_n = C_{n-1} - 1$

Lesson 1.3

3. a. A and B

4. a. A

b. B

8. a. A

b. C

9. a. From first to last: A, B, C, D.

b. From first to last, the new ranking is: B, A, D.

11. a. There are two new comparisons. A total of three comparisons must be made.

    b. There are three new comparisons. A total of six comparisons must be made.

    c.

1	0	0
2	1	1
3	2	3
4	3	6
5	4	10
6	5	15

Lesson 1.4

1. Nondictatorship.

2. If the method were repeated, the same ranking might not result. Therefore, condition 5 is violated. Nondictatorship (condition 1) is also violated.

4. Condition 4.

7. None.

12. a. { } {A} {B} {C} {A, B} {A, C} {B, C} {A, B, C}

    b. { } {A} {B} {C} {D} {A, B} {A, C} {A, D} {B, C} {B, D} {C, D} {A, B, C} {A, B, D} {A, C, D} {B, C, D} {A, B, C, D}

    c. $V_n = 2V_{n-1}$

14. 4 5

15. $V1_n = V1_{n-1} + 1$ or $V1_n = \frac{n}{n-1} V1_{n-1}$

Lesson 1.5

1. a. The possible coalitions: { ; 0} {A; 3} {B; 2} {C; 1} {A, B; 5} {B, C; 3} {A, C; 4} {A, B, C; 6}.
   The winning coalitions: {A, B; 5} {A, C; 4} {A, B, C; 6}.

   b. A:3 B:1 C:1

   c. A:2 B:2 C:0

6. $C_n = 2C_{n-1}$

7. { } {A} {B} {C} {D} {A, B} {A, C} {A, D} {B, C} {B, D} {C, D} {A, B, C} {A, B, D} {A, C, D} {B, C, D} {A, B, C, D}.

8. a. The winning coalitions are: {A, B; 51%} {A, C; 51%} {A, B, C; 76%} {A, B, D; 75%} {A, C, D; 75%} {B, C, D; 74%} {A, B, C, D; 100%}. Of these, A is essential to 5, B to 3, C to 3, and D to 1.

   b. The winning coalitions are: {A, B; 88%} {A, C; 54%} {A, D; 52%} {A, B, C; 95%} {A, B, D; 93%} {A, C, D; 59%} {B, C, D; 53%} {A, B, C, D; 100%}.
   Of these, A is essential to 6, B to 2, C to 2, and D to 2.

   c. In part a, D has 24% of the stock and one-twelfth of the power. In part b, D has 5% of the stock, but two-twelfths of the power.

Chapter 1 Review

2. a. D
   b. B
   c. A
   d. E
   e. C
   f. C

3. 19, 42, 89

4. a. It can occur in either the runoff or sequential runoff method.

5. a. Wilson, no.
   b. They ranked him last.
   c. The voters in the last group could have switched to Roosevelt, their second choice, and thereby prevented Wilson from winning.
   d. Borda, runoff, and Condorcet give the election to Roosevelt.

6. Conditions 2, 3, and 5.

7. Arrow proved that no group ranking method that ranks three or more choices will always adhere to his five fairness conditions.

8. Yes.

9. a. Clinton: $.43 + .2 \times .38 + .35 \times .19 = .5725$ or about 57%
Bush: $.38 + .15 \times .43 + .3 \times .19 = .5015$, or about 50%
Perot: $.19 + .3 \times .43 + .2 \times .38 = .395$, or about 40%

11. a. {A, B, C, D; 12}, {A, B, C; 10}, {B, C, D; 8}, {A, C, D; 9}, {A, B, D; 9}, {A, B; 7}, {A, C; 7}

b. A: 5, B: 3, C: 3, D: 1

c. No, A's power is disproportionately high, while D's is low.

d. All voters now have equal power.

Chapter 2

Lesson 2.2

2. a. \$35,000 \$30,000

b. \$70,000 − \$35,000 = \$35,000

c. \$35,000

d. \$30,000 \$37,500 \$32,500

f. Marmaduke would receive \$35,000.

3. a.

	Amy	Bart	Carl
Final settlement	4,788.89	4,988.89	4,722.23

6. a. $\begin{bmatrix} 10{,}000 & 94{,}000 & 0 \\ 8{,}000 & 100{,}000 & 0 \\ 9{,}000 & 96{,}000 & 0 \end{bmatrix}$

b. The value to Alan of the items that Betty receives.

Lesson 2.3

1. b. 42.857

c. Sophomore quota: 10.83.

d. Sophomore seats: 11; junior seats: 6; senior seats: 4.

2. a. Sophomore adjusted ratio: 42.18; junior adjusted ratio: 40.

b. Sophomore seats: 11; junior seats: 6; senior seats: 4.

6. a. 4.545 4.762
10.454 10.952
When the ideal ratio is 22, the decimal part of the 100-member class is larger than the decimal part of the 230-member class. The situation is reversed when the ideal ratio drops to 21.

b. For a small class.

Lesson 2.4

1. a.

10.83	10	10	11	11
5.6	5	5	6	6
4.57	4	4	5	5

Sophomores: 11; juniors: 6; seniors: 4.

c. 43.63 43.82
43.56 43.83

d. 43.56 (Sr) 43.63 (Jr) 44.19 (So.)
Sophomore seats: 11; junior seats: 6; senior seats: 4.

e. 43.82 (Jr) 43.83 (Sr) 44.24 (So).
Sophomore seats: 11; junior seats: 5; senior seats: 5.

f. The method favored by a given class can be seen in the following table of final apportionment results:

	Hamilton	Jefferson	Webster	Hill
Sophomore	11	11	11	11
Junior	6	6	6	5
Senior	4	4	4	5

4. a. 50

b. 3.7 4
2.6 3
1.6 2

c. 52.8571
52
53.3333

d. Freshman: 21; sophomore: 4; junior: 3; senior: 2.

Lesson 2.5

1. Ann will feel she has exactly one-third. Bart and Carl each could feel he received more than one-third.

4. One-sixth or 0.16.

5. b. One-sixth.

c. Four-sixths or 0.67.

d. One-third or 0.33.

7. a. $2 \times 3 = 6$

b. $k(k + 1)$ or $k^2 + k$

8. a. Yes. No.

b. Yes. Yes.

c. Probably not. Yes.

Lesson 2.6

1. a. $k + 1$ $k - 1$

b. $k + 2$ k $2k + 1$ $2k - 1$

2. a. New handshakes: 3; total handshakes: 6.

b. New handshakes: 4; total handshakes: 10.

3. a. 7

b. k

c. $H_n = H_{n-1} + (n - 1)$

4. a. 45

b. $\frac{k(k-1)}{2}$ $\frac{2k(2k-1)}{2}$ $\frac{(k+1)k}{2}$

c. $\frac{k(k-1)}{2}$

d. $\frac{(k+1)k}{2}$

e. k

f. $\frac{k(k-1)}{2} + k = \frac{k^2 - k}{2} = \frac{2k}{2} = \frac{k^2 + k}{2} = \frac{(k+1)k}{2}$

6. a. $V_{k+1} = 2V_k$

b. $V_n = 2^n$

c. $V_{k+1} = 2^{k+1}$

Chapter 2 Review

2. Answers are rounded to the nearest dollar.

	Joan	Henry	Sam
Fair share	\$8,880	\$8,600	\$8,433
Items received	Lot	Computer, stereo	Boat
Cash	\$ 880	\$6,000	\$1,733
Final settlement	\$9,342	\$9,062	\$8,895

3. Answers are rounded to the nearest dollar.

	Anne	Beth	Jay
Fair share	\$1,800	\$1,567	\$1,617
Items received	Car, computer		Stereo
Cash	−\$2,600	\$1,567	\$ 417
Final settlement	\$2,005	\$1,772	\$1,822

4. a. 10

b. 64.7, 24.7, 10.6

c. 65, 25, 10

d. 64, 24, 10

e. 9.9538, 9.8800, 9.6364

f. 65, 25, 10

g. 65, 25, 11

h. 10.0310, 10.0816, 10.0952

i. 64, 25, 11

j. 65, 25, 11

k. 10.0313, 10.0837, 10.1067

l. 64, 25, 11

m. 64, 25, 11

n. State A gained population and State C lost, but A lost a seat to C.

5. Balinski and Young proved that any apportionment method will sometimes produce one of three undesirable results: violation of quota, the loss of a seat when the size of the legislative body is increased even if population doesn't decrease, and the loss of a seat by one state whose population has increased to another whose population has decreased.

6. Arnold and Betty.

7. Have each of the original four divide his or her piece into five pieces that he or she considers equal. Have the new person select a piece from each of the others.

8. $\frac{k^2 - k}{2}$ or $\frac{k(k-1)}{2}$

Chapter 3

Lesson 3.1

3.

	Jackets	Shirts	Pants
Boutique 1	25	75	75
Boutique 2	30	50	50
Boutique 3	20	40	35

4. a. A_{21} = \$1.09, A_{12} = \$10.86, A_{32} = \$3.89

   b. A_{21} represents the cost of drinks at Vin's.
   A_{12} represents the cost of pizza at Toni's.
   A_{32} represents the cost of salad at Toni's.

   c. S_3 represents the cost of pizza at Sal's.

8. $$A + B = \begin{bmatrix} 10.10 + 1.15 & 10.86 + 1.10 & 10.65 + 1.25 \\ 3.69 + 0.00 & 3.89 + 0.45 & 3.85 + 0.50 \end{bmatrix}$$

$C =$

	Vin's	Toni's	Sal's
Pizza	\$11.25	\$11.96	\$11.90
Salad	\$3.69	\$4.34	\$4.35

12. Decrease in batting average is shown using a negative sign.

Lesson 3.2

1. a. T represents the cost of four pizzas with additional toppings and four salads with a choice of two dressings from each of the three pizza houses.

b. $47.60

c. T_{12} represents the cost of four pizzas with two toppings at Toni's.

d. T_{21} represents the cost of four salads with choice of two dressings at Vin's.

2. a., b.

$$J = \begin{array}{c} \\ \text{Pearl} \\ \text{Jade} \end{array} \begin{array}{c} \begin{array}{cccc} \text{e} & \text{p} & \text{n} & \text{b} \end{array} \\ \begin{bmatrix} 16 & 8 & 12 & 10 \\ 40 & 20 & 24 & 18 \end{bmatrix} \end{array}$$

c. 24.

d. J_{21} represents the number of jade earrings that Nancy expects to sell in June.

e. J_{12} represents the number of pearl pins that Nancy expects to sell in June.

8.

$$\begin{array}{c} \\ \text{Rate} \end{array} \begin{array}{c} \begin{array}{ccc} \text{CD} & \text{CU} & \text{Bond} \end{array} \\ [0.073 \quad 0.065 \quad 0.075] \end{array} \begin{array}{c} \text{CD} \\ \text{CU} \\ \text{Bd} \end{array} \begin{array}{c} \$ \\ \begin{bmatrix} 10{,}000 \\ 17{,}000 \\ 12{,}000 \end{bmatrix} \end{array} = \$2{,}735$$

9. a. The transpose of a row matrix is a column matrix and the transpose of a column matrix is a row matrix.

b.

$$M^T = \begin{array}{c} \\ \text{e} \\ \text{p} \\ \text{n} \\ \text{b} \end{array} \begin{array}{c} \begin{array}{cc} \text{p} & \text{j} \end{array} \\ \begin{bmatrix} 8 & 20 \\ 4 & 10 \\ 6 & 12 \\ 5 & 9 \end{bmatrix} \end{array}$$

c. Answers will vary. A possible answer: It may be necessary to use the transpose of a matrix when performing matrix multiplication. (See Exercise 10.)

10. a.

	e	p	n	b
hours	2	1	2.5	1.5

b.

	e	p	n	b
hours	2	1	2.5	1.5

	p	j
e	8	20
p	4	10
n	6	12
b	5	9

c.

	p	j
hours	42.5	93.5

d. It takes Nancy 42.5 hours to make the pearl jewelry and 93.5 hours to make the jade jewelry.

Lesson 3.3

1. a.

	Mike	Liz	Kate
Mike	$261,000	0	$250
Liz	$235,000	0	$215
Kate	$255,000	0	$325

b. The entries in row 1 represent the value to Mike of the items that he, Liz, and Kate received. Row 2 represents the values to Liz, and row 3 the values to Kate.

2. a. Matrix Q:

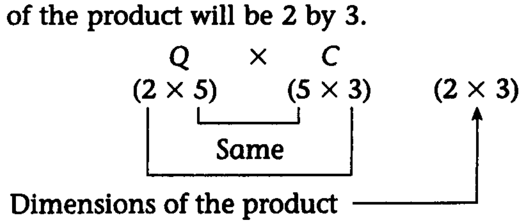
	Burger	Special	Potato	Fries	Shake
Emma	0	1	0	1	1
Ken	1	0	1	0	1

b. Matrix C:

	Cal.	Fat	Chol.
Burger	450	40	50
Special	570	48	90
Potato	500	45	25
Fries	300	30	0
Shakes	400	22	50

c. The dimensions of Q are 2 by 5, and those of C are 5 by 3.

d. The dimensions of the product will be 2 by 3.



e. The dimensions of C can be described as Foods by Contents and the dimensions of Q times C can be described as Persons by Contents.

Persons by Foods × Foods by Contents = Persons by Contents

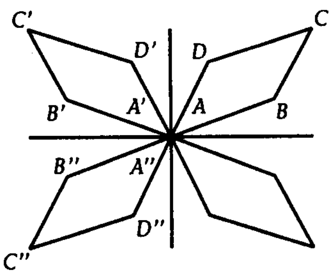
Same

Dimensions of the product

f.

	Cal.	Fat	Chol.
Emma	1,270	100	140
Ken	1,350	107	125

8. The diagram below shows the polygons plotted for parts a through h.



a. $T_1P = \begin{bmatrix} 0 & -6 & -8 & -2 \\ 0 & 2 & 6 & 4 \end{bmatrix}$

c. Polygon $A'B'C'D'$ is the reflection of polygon $ABCD$ in the y-axis.

d. $T_2P = \begin{bmatrix} 0 & -6 & -8 & -2 \\ 0 & -2 & -6 & -4 \end{bmatrix}$

e. Polygon $A''B''C''D''$ is the reflection of polygon $A'B'C'D'$ in the x-axis.

f. $RP = \begin{bmatrix} 0 & -6 & -8 & -2 \\ 0 & -2 & -6 & -4 \end{bmatrix}$

The effect of R on P is to rotate $ABCD$ 180 degrees about the origin.

g. $T_3 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$T_3P'' = \begin{bmatrix} 0 & 6 & 8 & 2 \\ 0 & -2 & -6 & -4 \end{bmatrix}$

h. $$T_4 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = T_3T_2$$

$$T_4P' = \begin{bmatrix} 0 & 6 & 8 & 2 \\ 0 & -2 & -6 & -4 \end{bmatrix}$$

Lesson 3.4

1. a. 18.97

b. 9.96, 8.1, 7.29, 9.36, 2.4

c. 18.97, 9.96, 8.1, 7.29, 9.36, 2.4; total 56.

d. 9 months: 18.32, 11.38, 8.96, 7.29, 5.83, 5.62; total 57.
12 months: 18.02, 10.99, 10.24, 8.06, 5.83, 3.5; total 57.

e. Answers may vary. A possible answer: The population continues to grow. The rate of growth seems to have slowed.

f. Answers may vary. A possible answer: The population growth may continue to slow or even become constant.

2. a. 118.4

b. The product is the number of newborn deer after 1 cycle.

c. 30, 24, 21.6, 21.6, 8.4

d. i. $[50\ 30\ 24\ 24\ 12\ 8] \begin{bmatrix} 0.6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ ii. $\begin{bmatrix} 0 \\ 0.8 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

iii. $\begin{bmatrix} 0 \\ 0 \\ 0.9 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ iv. $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.9 \\ 0 \\ 0 \end{bmatrix}$ v. $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.7 \\ 0 \end{bmatrix}$

3. a. Distribution after 3 months: 0, 21, 0, 0, 0, 0; total 21.

b. Distribution after 3 months: 11, 3, 4.5, 4.5, 4, 3; total 30.

Lesson 3.5

1. a. P_5 = [19.47 10.81 9.89 9.22 6.45 3.50]

   b. Total: 59.35

   c. P_7 = [20.47 12.11 10.51 8.76 7.12 4.43]; total: 63.41.

2. To reach 250 females:

	Cycles	Population	Years
a.	61	253.2	15.25
b.	69	250.9	17.25
c.	76	252.9	19.00
d.	42	257.2	10.5

3. a. 4 cycles, 56.65, −1.31%
   5 cycles, 59.35, 4.77%
   6 cycles, 61.76, 4.06%

   b. Population appears to decline, then increase again.

   c. P_{25} = 108.488, P_{26} = 111.789, P_{27} = 115.191; growth rates: 3.04%, 3.04%.

4. a. The long-term growth rate of the total population is 3.04% in each case.

   b. The initial population does not effect the long-term growth rate.

Chapter 3 Review

2. No. Answers will vary. Possible answer: The matrices must be conformable.

3. a.

$$L = \begin{matrix} M & C & S & D \\ [35 & 6 & 6 & 12] \end{matrix}$$

   b. L_2 = number of bags of chips ordered
   L_4 = number of six-packs of drinks

   c. $216.60

4. a.

	Lodging	Food	Rec.
Crystal	13.00	20.00	5.00
Springs	12.50	19.50	7.50
Bear	20.00	18.00	0.00
Beaver	40.00	0.00	0.00

b. $C_{22} = 19.50$
$C_{43} = 0$

c. C_{13} = cost for recreation at Crystal Lodge
C_{31} = cost for lodging at Bear Lodge

5. a.

	System	Cart.	Case
Z-Mart	39.50	24.50	8.50
Base	49.90	29.95	12.50

b.

	System	Cart.	Case
Z-Mart	35.55	22.05	7.65
Base	39.92	23.96	10.00

c.

	System	Cart.	Case
Z-Mart	3.95	2.45	0.85
Base	9.98	5.99	2.50

d.

	System	Cart.	Case
Z-Mart	142.20	88.20	30.60
Base	159.68	95.84	40.00

6. a.

	Plate	Large	Small
No.	5	3	7

b.

	Ebony	Walnut	Rose	Maple
Plate	100	800	600	400
Large	200	1200	1000	800
Small	50	500	450	400

c.

Ebony	Walnut	Rose	Maple
1450	11,000	9,150	7,200

d.

	Plate	Large	Small
No.	5	3	7

	Weeks
Plate	3
Large	4
Small	2

= No. [15 + 12 + 14] (Weeks)
= [41]

7.
$$\begin{array}{ccc} \text{Tennis} & \text{Golf} & \text{Soccer} \\ [\$50{,}000 & \$100{,}000 & \$75{,}000] \end{array} \begin{array}{l} \\ \text{Tennis} \\ \text{Golf} \\ \text{Soccer} \end{array} \begin{array}{c} \text{Return} \\ \begin{bmatrix} 0.082 \\ 0.065 \\ 0.075 \end{bmatrix} \end{array} = \begin{array}{c} \text{Return} \\ [\$16{,}225] \end{array}$$

8.
$$\begin{array}{ccc} \text{Jazz} & \text{Symp.} & \text{Orch.} \\ \$[300.00 & 335.00 & 373.50] \end{array}$$

9. $A^T = \begin{bmatrix} 4 & 5 \\ 2 & 1 \\ 6 & 3 \end{bmatrix}$

10. a. 3×2

 b. 4×3

 c. Not possible.

 d. 4×3

11. a. $M^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ $M^3 = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$ $M^4 = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}$

 b. $M^5 = \begin{bmatrix} 16 & 16 \\ 16 & 16 \end{bmatrix}$

 c. $M^n = \begin{bmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{bmatrix}$

 e. $M^2 = \begin{bmatrix} 1 & 0 \\ 8 & 9 \end{bmatrix}$ $M^3 = \begin{bmatrix} 1 & 0 \\ 26 & 27 \end{bmatrix}$ $M^4 = \begin{bmatrix} 1 & 0 \\ 80 & 81 \end{bmatrix}$

 $M^5 = \begin{bmatrix} 1 & 0 \\ 242 & 243 \end{bmatrix}$ $M^n = \begin{bmatrix} 1 & 0 \\ 3^n - 1 & 3^n \end{bmatrix}$

12. Identity, a square matrix with ones along the diagonal and zeros elsewhere.

13. a. Yes. $AB = BA = I$

 b. Yes. $AB = BA = I$

 c. No. Not square matrices.

14. a.

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \text{Male} \quad \text{Female} \\ \begin{bmatrix} 189 & 196 \\ 175 & 180 \\ 251 & 254 \end{bmatrix} \end{array}$$

b. $\begin{array}{c} A \\ B \\ C \end{array} \begin{bmatrix} 385 \\ 355 \\ 505 \end{bmatrix}$

15. a. 24 months.

b.

$$L = \begin{bmatrix} 0.0 & 0.6 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.5 & 0.0 & 0.8 & 0.0 & 0.0 & 0.0 \\ 1.1 & 0.0 & 0.0 & 0.9 & 0.0 & 0.0 \\ 0.9 & 0.0 & 0.0 & 0.0 & 0.8 & 0.0 \\ 0.4 & 0.0 & 0.0 & 0.0 & 0.0 & 0.6 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

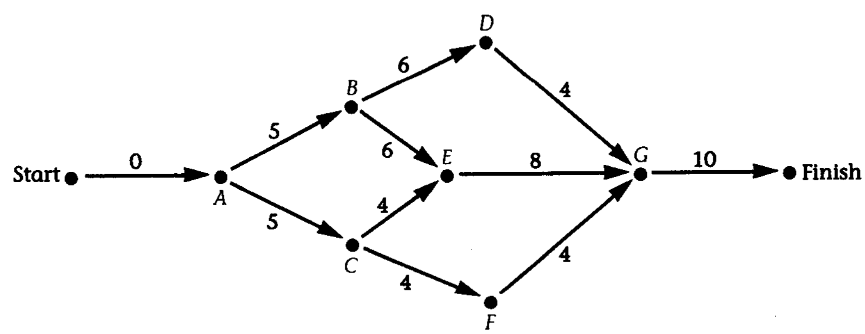
c. 10%

d. 28 months.

Chapter 4

Lesson 4.1

1.



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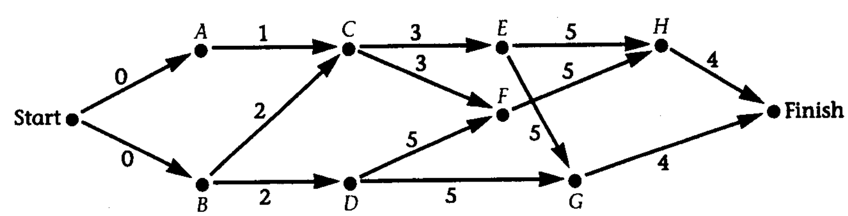


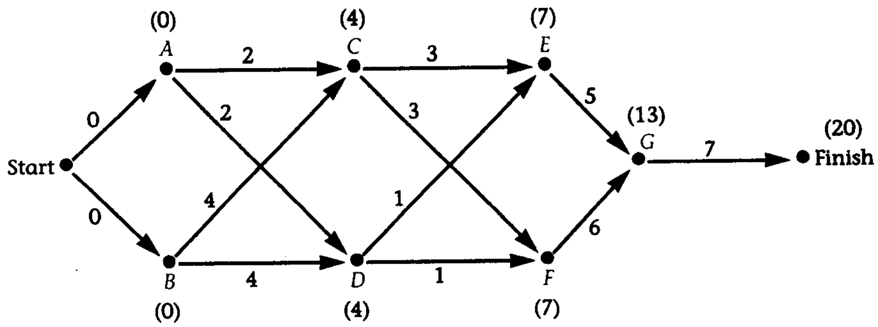
7. a. A: 2, none; B: 4, none; C: 3, A; D: 3, C and B; E: 2, B; F: 1, E; G: 4, D and F.

Lesson 4.2

1. EST for *C* through *G*: 7, 10, 11, 16, 23.
   Min. project time: 26.
   Critical path: Start—*ACEFG*—Finish.

3. EST for *D* through *I*: 6, 9, 8, 13, 11, 18.
   Min. project time: 26.
   Critical path: Start—*CFI*—Finish.

5. a.



   b. 22

   c. Start—*ADG*—Finish.

   d. The minimum time is reduced to 21 days, to 20 days.

   e. No, below 8 days *A* is no longer on the critical path.

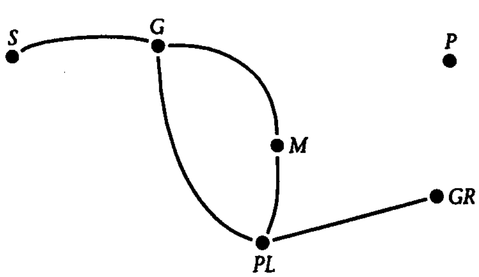
8. a. Day 16, day 17, day 18, both task *G* and the project will be delayed.

   b. Day 11.

   c. Day 5, day 6, day 5.

Lesson 4.3

1.

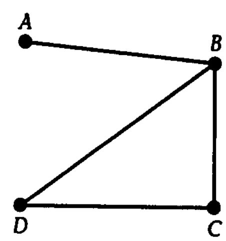


3. a. *A* and *F*, *B* and *F*, *B* and *C*, *A* and *C*, *A* and *D*, *A* and *E*, *B* and *E*, *B* and *D*, *F* and *C*, or *F* and *D*.

   b. *FEDC*.

   c. No, there is no path from *A* or *B* to the vertices *C*, *D*, *E*, or *F*.

   d. No, not every pair of vertices is adjacent. For example, *B* and *C* are not adjacent.

5. 

6. $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

9. $V{:}3$, $X{:}2$, $Y{:}2$, $Z{:}1$

10. a. $B{:}2$, $C{:}6$, $D{:}3$, $E{:}2$

    b. $\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix}$

11. 4, 5, 6; 12, 20, 30;
    $T_4 = T_3 + 6$ $\quad T_5 = T_4 + 8$
    $T_6 = T_5 + 10$ $\quad T_n = T_{n-1} + 2(n-1)$

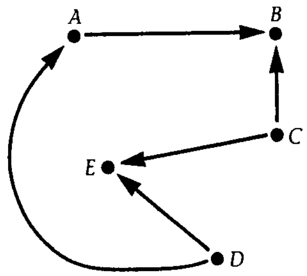
Lesson 4.4

1. a. Both, the degrees of all vertices are even.

2. New circuit: *S, b, e, a, g, f, S.*
   Final circuit: *S, e, f, a, b, c, S, b, e, a, g, f, S.*

3. Answers may vary. One possible circuit: *e, d, f, h, d, c, h, b, c, g, a, h, g, f, e.*

8. a.



9. a. Yes. b. No. c. Yes.

11. a.

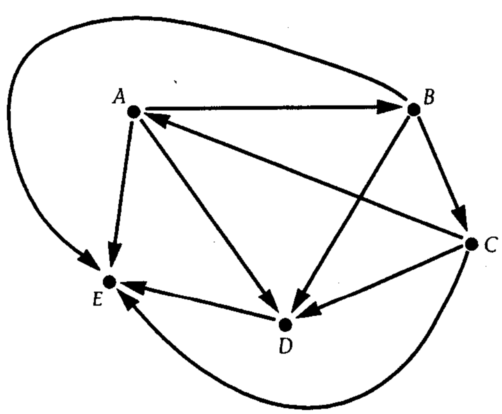
	a	*b*	*c*	*d*	*e*
a	0	0	1	0	1
b	1	0	0	1	0
c	0	1	0	0	0
d	0	0	1	0	0
e	0	1	0	1	0

Lesson 4.5

1. a. Yes.

b., c. The theorem does not apply.

8.



11. 6, 10, 15 $S_n = S_{n-1} + (n - 1)$

15. a.

$$M = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

b.

$$M^2 = \begin{bmatrix} 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

c. The winner would be B.

Lesson 4.6

1. 4, 3, 2

3. a. List the vertices in order from the ones with the greatest degree to the ones with the least.

b. Those not adjacent to it or adjacent to one with that color.

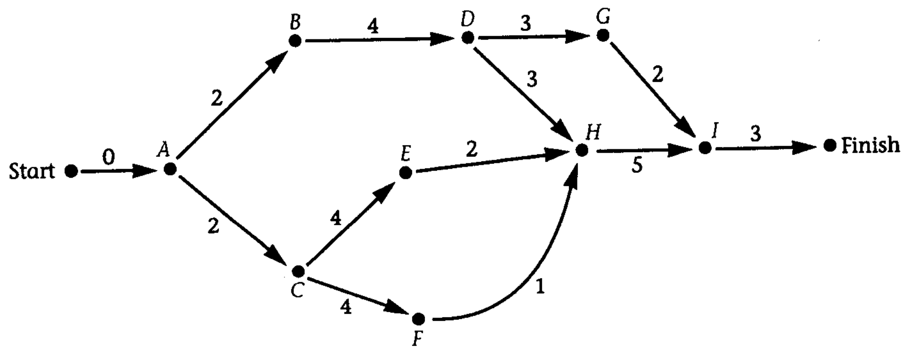
5. a. 2, 3, 4, 5

9. 4

12. a.

Chapter 4 Review

2.



3.

Task	Time	Prerequisites
Start	0	—
A	2	None
B	3	None
C	4	*A*
D	4	*A*, *B*
E	2	*B*
F	3	*C*
G	5	*D*, *E*
H	7	*F*, *G*
Finish		

4. a. *A*, (0); *B*, (4); *C*, (4); *D*, (7); *E*, (6); *F*, (11); *G*, (10); *H*, (15); *I*, (16); *J*, (18).

b. Minimum project time: 23.

5. a. *A*, (0); *B*, (2); *C*, (2); *D*, (6); *E*, (6); *F*, (6); *G*, (9); *H*, (9); *I*, (14).

b. Critical path: Start—*ABDHI*—Finish.
Minimum project time: 17.

6. a. Yes, a path exists from each vertex to every other vertex.

b. No, not every pair of vertices is adjacent.

c. *A*, *D*, or *C*.

d. *BCDE* or *BCAE*.

e. Deg(C) = 4.

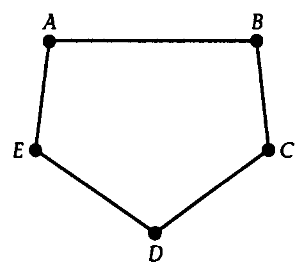
f.

	A	B	C	D	E
A	0	0	1	0	1
B	0	0	1	0	0
C	1	1	0	1	1
D	0	0	1	0	1
E	1	0	1	1	0

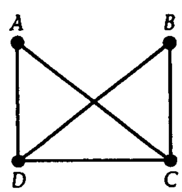
7. a. Euler path. Two vertices have odd degrees and the remaining vertices have even degrees.

b. Euler circuit. All vertices have even degrees.

8. a.



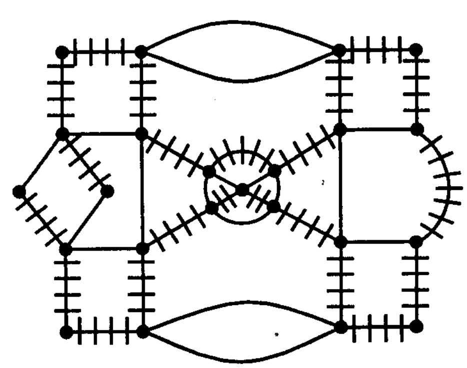
b.



9. a. No.

b. Yes.

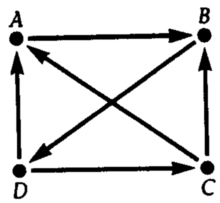
c.



10. a. Yes.

b. No, the graph has exactly two odd vertices. You would have to begin at one of the vertices with an odd degree and end at the other.

11. a.

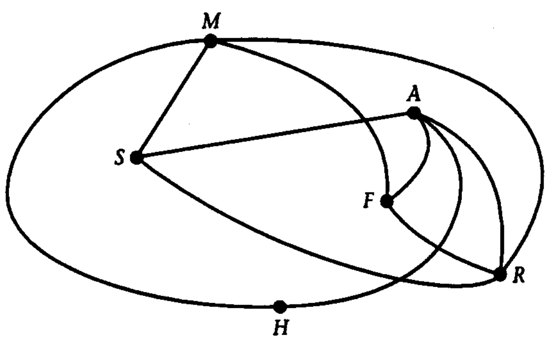


b. There is no Condorcet winner because none of the candidates can beat all of the other candidates in a one-on-one race.

c. *D, C, A, B*; *C, A, B, D*; *C, B, D, A*; *B, D, C, A*; *A, B, D, C*.

d. If the Hamiltonian path, *B, D, C, A* is chosen for a pairwise voting scheme, *B* will win. The path shows that for *B* to "survive," you need first to pair *A* and *C*. *C* wins. Next pair the winner, *C*, against *D*, and *D* wins. Finally, pair the winner, *D*, against *B* and *B* is the final winner of the election.

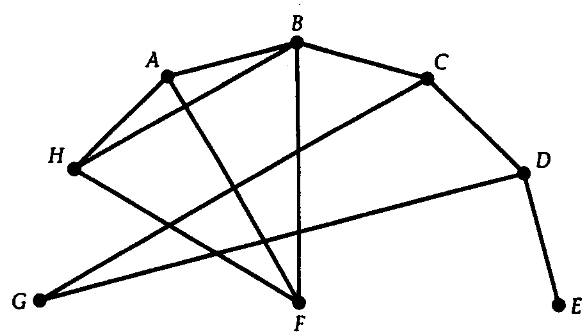
12. a.



b. Three time slots. One possible schedule:

Time 1—Math and Art Time 2—Reading and History
Time 3—Science and French

13. a.



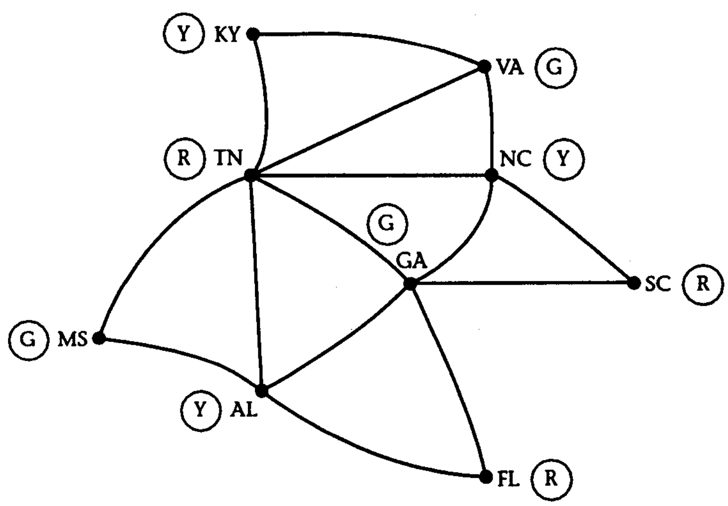
b. Four frequencies.

14. a. No, the outdegrees are not equal to the indegrees at each vertex.

b. Yes, the outdegree equals the indegree at all vertices but two. At one of those two vertices, the indegree is one greater than the outdegree and at the other vertex, the outdegree is one greater than the indegree.

Answers will vary, but the paths must begin at *B* and end at *D*.

15. a., b.

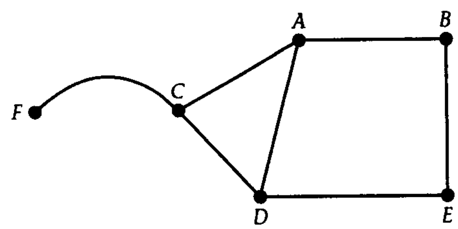


c. Three colors.

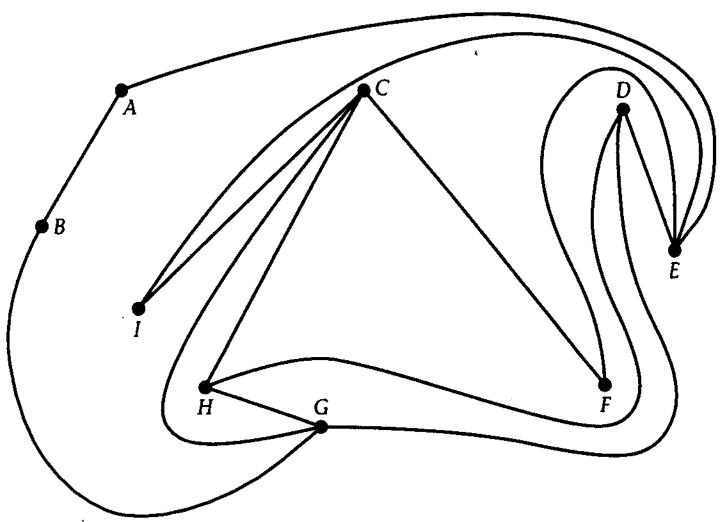
Chapter 5

Lesson 5.1

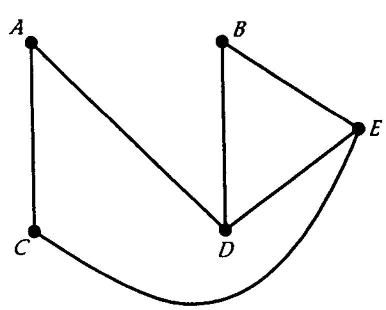
1. Planar.



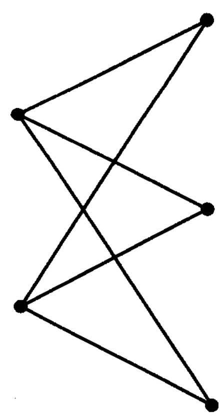
4. Hint—move A and B around.



7.



10. a.



$K_{2,3}$

11. a. {*A*, *B*, *C*, *D*, *E*, *F*} and {*G*}

14. 6, 12, $m * n$

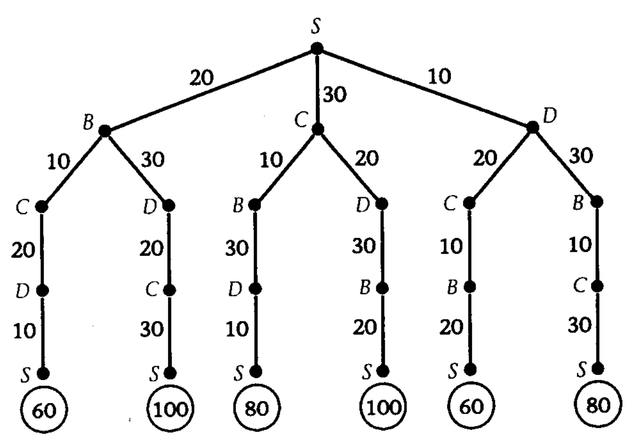
17. 30 handshakes, bipartite.

20. a and d, because all of the edges and vertices of the original graph are in a and d.

22. No. No.

Lesson 5.2

1. a., b.

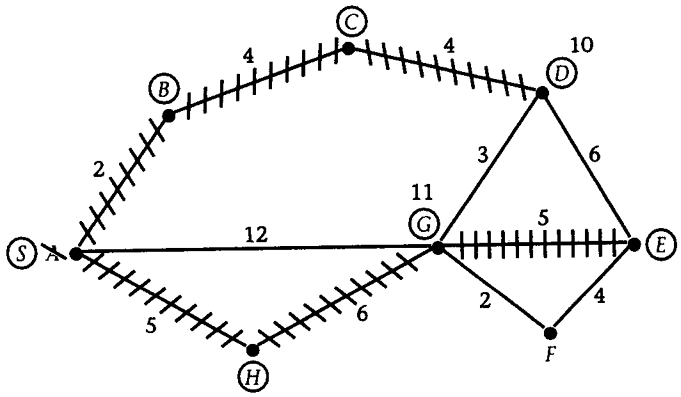


c. *SDCBS* d. *SDCBS* e. Yes.

5. a. 0.36 seconds, about 24 hours. b. 3.6×10^{-7} seconds, 0.09 seconds, about 676 hours.

Lesson 5.3

1. 6, 12, 11, *C*, *BC*; the shortest path from *A* to *E* is *AHGE*.

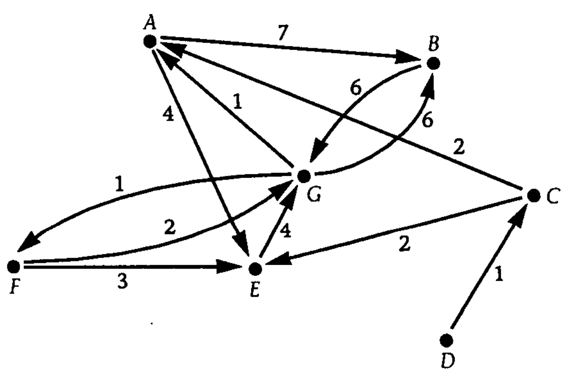


2. *ABECDF* (11)

5. a. Albany, *CEH*, Ladue

b. Albany, *BD*, Fenton, *GK*, Ladue. This problem yields a different solution than part a because you have to find two solutions and then add them: first you must find the shortest path from Albany to Fenton, then a path with Fenton as Start.

8.

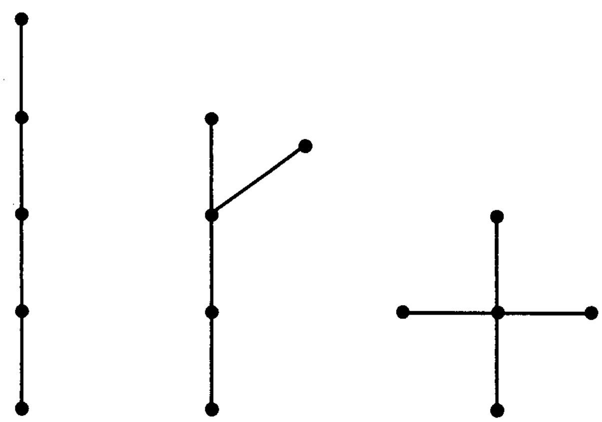


Shortest path: *DCAB*; least charge: 1 + 2 + 7 = 10.

Lesson 5.4

1. *BCEFB, CDEC, BCDEFB, BCFB, CEFC, CDEFC*

3. Five vertices.



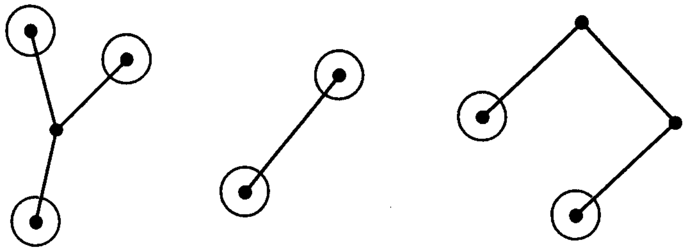
4.

Number of vertices	Number of edges
1	0
2	1
3	2
4	3
n	$n - 1$

a. 18 edges b. 16 vertices
c. The number of vertices = the number of edges −1.

7.

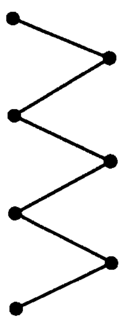
1	0	$S_1 = 0$
2	2	$S_2 = S_1 + 2$
3	4	$S_3 = S_2 + 2$
4	6	$S_4 = S_3 + 2$
5	8	$S_5 = S_4 + 2$
6	10	$S_6 = S_5 + 2$
		$S_n = S_{n-1} + 2$

9. 

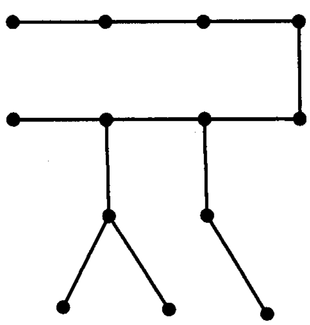
11. Rhombus.

Lesson 5.5

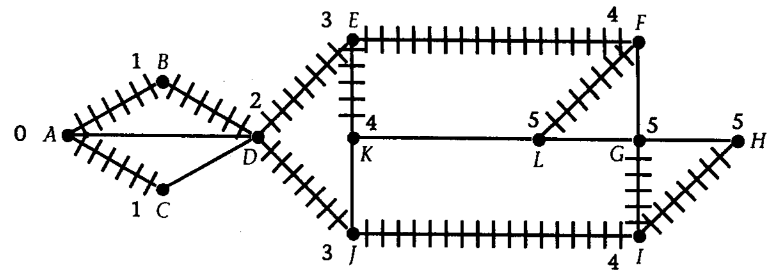
1. One possible spanning tree:



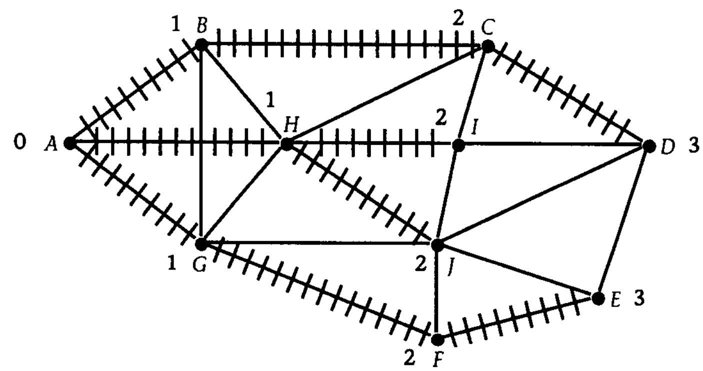
4. One possible spanning tree:



6. a. Yes. b. *E* and *J* c. *DE* and *DJ* d. *F, K, I*



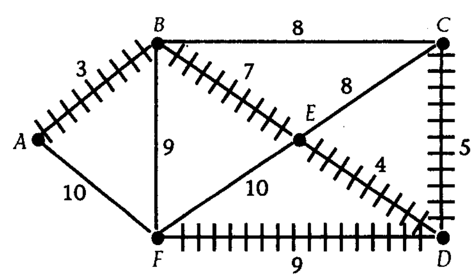
7. This is one of many possibilities.



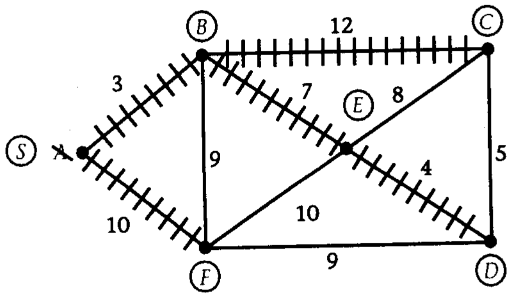
10. The minimum weight is 10.

13. $2,100

17. a. Weight 28.



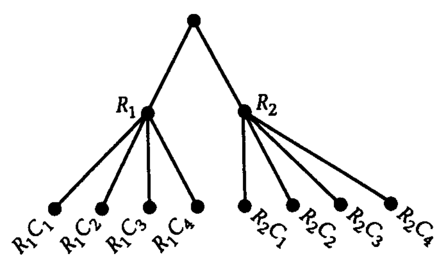
b. A to F is 10; A to B is 3; A to C is 15; A to E is 10; A to D is 14.



c. No. It is a spanning tree, but in this example, it is not minimal. Its total weight is 36, which is greater than 28.

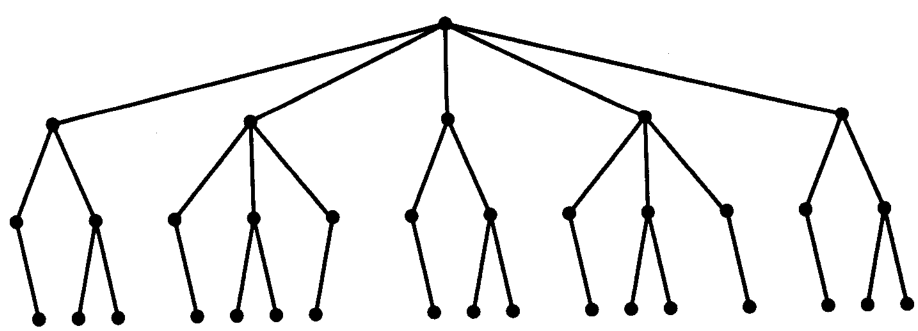
Lesson 5.6

1.

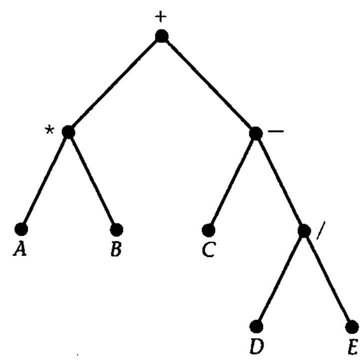


3. Binary tree. a. V is level 2. b. C is the parent. c. G and H are children.

7. There are 17 questions in the book.



11.



12. 3 2 * 8 2 3 * − +

15. a. 33 b. 7 c. 9 d. 14

16. a. 2 3 6 * + 4 1 + −

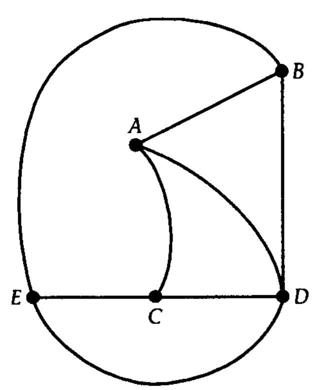
19. *ABDEGHCFI*

21. 17

22. a. 8

Chapter 5 Review

2.

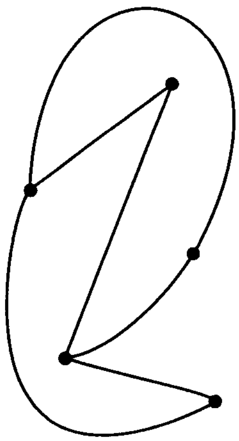


3. 2, 2, 2, 2

4. a. The vertices of the graph can be divided into two sets so that each edge of the graph has one endpoint in each set.

b. Yes, all possible edges from one set of vertices to the other are drawn.

c. Yes.



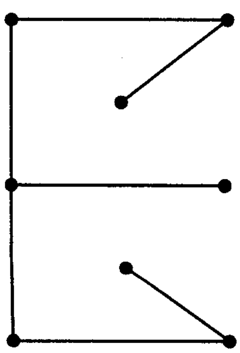
d. 2.

5. a. O–SCM–O.

b. 314 ft.

c. Hamiltonian circuit.

6.

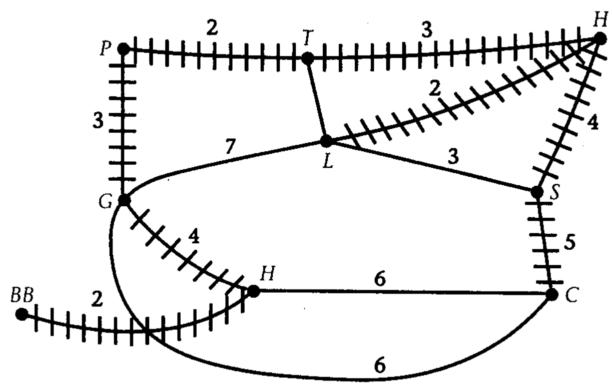


7. 3, 4, 5, n

8. a. Home–*T*–*P*–*G*–*H*–*BB*.

b. 14 miles.

c.



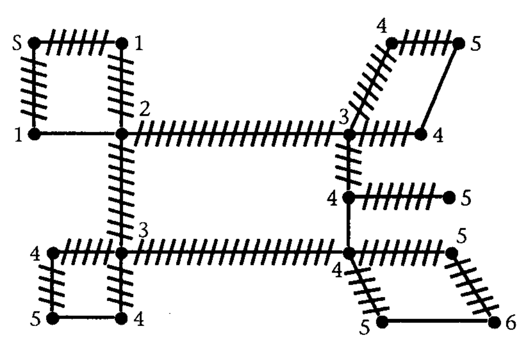
9. The total weight of a minimum spanning tree for the graph is 23 miles.

10. a. Yes, it is a connected graph with no cycles.

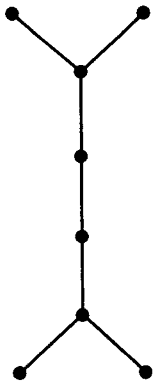
b. Yes, it is a connected graph with no cycles.

c. No, the graph contains a cycle.

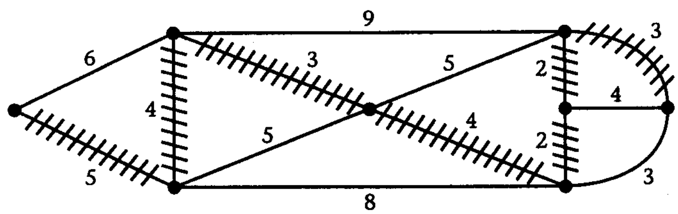
11.



12. One possible solution:



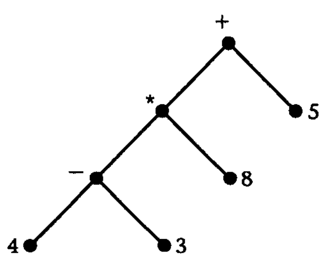
13. a.



b. Total cost = $23,000.

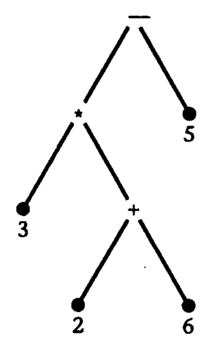
14. Problems similar to those in Lesson 5.5.

15.



16. 18

17. Any expression is possible. The following is just one example:
The expression: 3 * (2 + 6) − 5
The expression tree:



The postorder listing: 3 2 6 + * 5 −

Chapter 6

Lesson 6.1

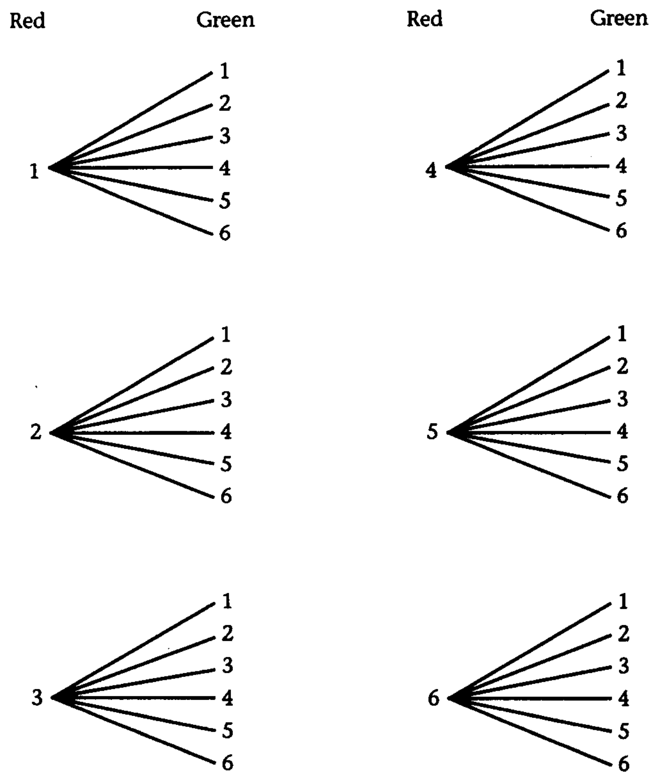
1. li, lo, ln, ls, il, io, in, is, ol, oi, on, os, nl, ni, no, ns, sl, si, so, sn.

2. 5, 4, $5 \times 4 = 20$, $6 \times 5 = 30$

3. (1,2) (1,3) (1,4) (1,5) (1,6) (1,7) (1,8) (1,9) (2,3) (2,4) (2,5) (2,6) (2,7) (2,8) (2,9) (3,4) (3,5) (3,6) (3,7) (3,8) (3,9) (4,5) (4,6) (4,7) (4,8) (4,9) (5,6) (5,7) (5,8) (5,9) (6,7) (6,8) (6,9) (7,8) (7,9) (8,9)

4. 9, 8, $9 \times \frac{8}{2} = 36$

13.



15. a. Win \$1: 10 ways; win \$2: 1 way; lose \$1: 25 ways.

Lesson 6.2

1. $\frac{10!}{6!}$ 5040

3. a. 15

b. A particular front sprocket and a particular rear sprocket.

5. a. 12,167,000

b. $\frac{1}{12,617}$

7. a. 72

b. $\frac{1}{2}$

c. 0

d. 1

10. a. 2704

b. 2652

13. a. 30! or about 2.6525×10^{32}.

b. About 1.0827×10^{28}; the number of seating arrangements is about 24,500 times as large.

14. a. 6

b. A road from Claremont to Upland and a road from Upland to Pasadena.

c. $3 \times 3 = 9$

18. a. 100,000 manufacturers.

b. 100,000 products.

19. a. 1,048,576

b. About 10 years; about 262 feet.

Lesson 6.3

3. a. 672

b. 504

c. 2,380. They are the same.

4. a. 210

b. 210

c. 1,024

5. a. 1,326

b. 325

c. $\frac{325}{1,326}$, or about 0.245.

8. a. 10

b. 10

9. a. 7,059,052

b. About 245 weeks, or a little less than 5 years.

c. About 353 feet.

d. 13,983,816

e. $\frac{80{,}000}{13{,}983{,}816}$, or about .00572.

f. The probability of winning in Virginia is nearly twice as good.

10. a. $C(6,5) \times C(38,1) = 228$

b. 10,545

c. 168,720

14. 255

19. a. 7

b. 21

c. 28

e. $\frac{7}{28}$ or $\frac{1}{4}$

f. 91

Lesson 6.4

1. a. $\frac{520}{1{,}000} = .52$

b. $\frac{196}{360}$, or about .544.

c. No, but they are fairly close.

d. $\frac{360}{1{,}000} = .36$

e. $\frac{196}{1{,}000} = .196$. The product is .1872.

f. .544 × .36 = .196. They are the same.

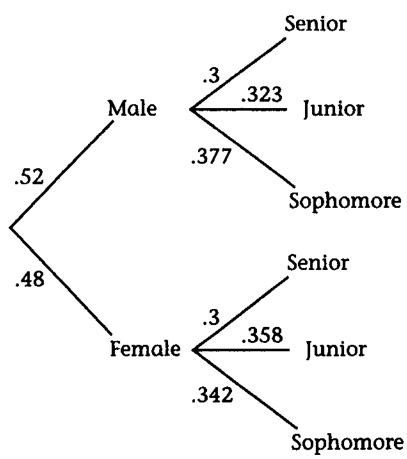
g. No.

3. a. $\frac{684}{1{,}000} = .684$

b. .36 + .52 = .88, which is larger than .684.

c. No.

5. a.



p(Male and Senior) = .52 × .3 = .156

p(Male and Junior) = .52 × .323 = .168

p(Male and Sophomore) = .52 × .377 = .196

p(Female and Senior) = .48 × .3 = .144

p(Female and Junior) = .48 × .358 = .172

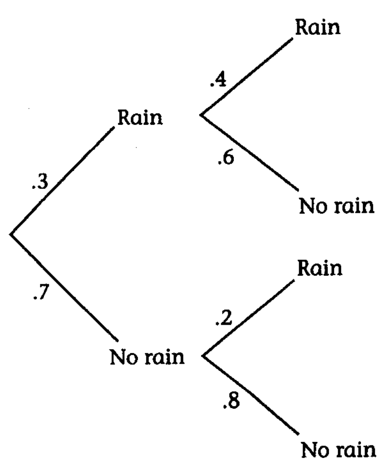
p(Female and Sophomore) = .48 × .342 = .164

b. 1

9. a. .54

b. That the outcomes of the two games are independent.

10. a.



p(Rain and Rain) = .3 × .4 = .12

p(Rain and No rain) = .3 × .6 = .18

p(No rain and Rain) = .7 × .2 = .14

p(No rain and No rain) = .7 × .8 = .56

b. .12

c. 32

d. .56

e. No.

13. $\frac{1}{216}$

16. a. $.977^6$, or about .870

b. The first is .870, and the second is $1 - \frac{1}{6} = \frac{5}{6}$, or about .83.

c. .000529

d. .999471, .99683

19. a. $\frac{1}{80}$

b. That the two events are independent.

c. The probability of selecting a man with red hair is .1; the probability of selecting a man who owns a blue car is .125; and the probability of selecting a man who has red hair and owns a blue car is about .0124. The product of the first two probabilities is .0125, so the events are quite close to being independent.

20. a. 36

b. $\frac{1}{36}$

c. $\frac{1}{1,296}$

d. $\frac{35}{36}$

e. $\frac{1,225}{1,296}$

f. $\left(\frac{35}{36}\right)^{21}$, or about .5534.

Lesson 6.5

1. a. The probability of each outcome is $\frac{1}{2}$, and successive applications are independent.

b. The probability of choosing the third answer is $\frac{1}{2}$, while the probability of choosing each of the others is $\frac{1}{4}$; successive applications are independent.

c. Each possibility probably has a $\frac{1}{4}$ chance of occurring, but successive applications may not be independent if, for example, one finger is injured.

d. Each possibility has the same chance of occurring, and successive applications are independent.

2. a. 10

b. .3125

c. .03125, .15625, .3125, .3125, .15625, .03125

d. .00243, .02835, .1323, .3087, .36015, .16807

3. a. .2051

b. .1172

c. .0439

d. .0098

e. .00098

f. .3770

5. a. .2765

b.

Number of women	0	1	2	3	4	5	6
Probability	.0466	.1866	.3110	.2765	.1382	.0369	.0041

8. a. $\frac{1}{7{,}059{,}052}$; $\frac{7{,}059{,}051}{7{,}059{,}052}$

b. $2.82, but this assumes that the jackpot is not shared with another party.

a.

Amount won	27,000,000	−5,000,000
Probability	$\frac{5{,}000{,}000}{7{,}059{,}052}$	$\frac{2{,}059{,}052}{7{,}059{,}052}$

The expectation is $19,124,380, but this assumes the jackpot is not shared.

10. a.

Amount won	−1	1	20
Probability	$\frac{21}{36}$	$\frac{14}{36}$	$\frac{1}{36}$

b. $0.36

c. No, the council would lose about 36 cents per play. One way of correcting this would be to give no prize for matching a single number. In fact, the jackpot could then be increased but should be kept under $35.

13. a. $\frac{1}{4}$

b.

Amount won	−$0.50	$1.00
Probability	$\frac{3}{4}$	$\frac{1}{4}$

c. −$0.50

d. Yes, about 50 cents per play.

Chapter 6 Review

2. a. $\frac{7{,}900}{46{,}900}$, or about .168.

b. $\frac{2{,}300}{13{,}700}$, or about .168.

c. $\frac{13{,}700}{46{,}900}$, or about .292.

d. $\frac{2{,}300}{46{,}900}$, or about .049.

e. $\frac{19{,}300}{46{,}900}$, or about .412.

f. Yes.

g. No.

3. $C(5,1) = 5$, $C(5,2) = 10$, $C(5,3) = 10$, $C(5,4) = 5$, $C(5,5) = 1$

4. a. $\frac{1}{16}$

b. $\frac{1}{16}$

c. $\frac{1}{8}$

5. a. 720

b. 48

c. $\frac{1}{15}$

d. The math books can be in positions 1 and 2, or in positions 2 and 3, or in positions 3 and 4, or in positions 4 and 5, or in positions 5 and 6. $5 \times 48 = 240$.

e. $\frac{1}{3}$

6. a.

Amount won	\$2	\$1	−\$1
Probability	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

b. $\frac{1}{4}$

c. Win about \$25.

7. 720, about 28

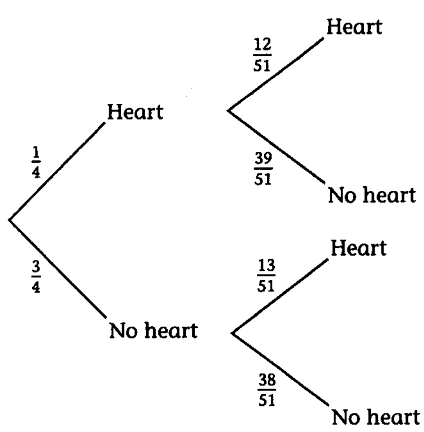
8. There are $C(39,5) = 575{,}757$ different winning tickets possible in the first and $C(36,6) = 1{,}947{,}792$ ways of winning in the second, so the probability of winning in the first is between three and four times as great as in the second. About five in the first and one or two in the second.

9. a. $\frac{1}{4}$

b. $\frac{4}{17}$

c. $\frac{1}{17}$

d.



$p(\text{Heart and Heart}) = \frac{1}{4} \times \frac{12}{51} = \frac{12}{204}$

$p(\text{Heart and No heart}) = \frac{1}{4} \times \frac{39}{51} = \frac{39}{204}$

$p(\text{No heart and Heart}) = \frac{3}{4} \times \frac{13}{51} = \frac{39}{204}$

$p(\text{No heart and No heart}) = \frac{3}{4} \times \frac{38}{51} = \frac{114}{204}$

e. No.

10. a. 165

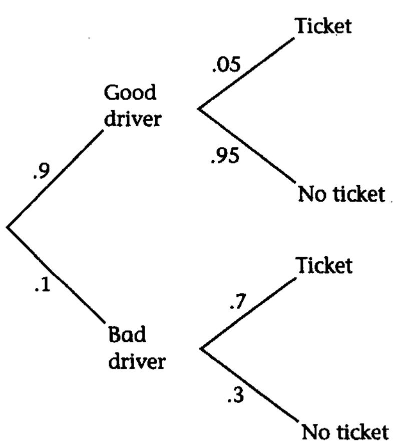
b. 75

c. $\frac{75}{165}$

d. 135

e. $\frac{135}{165}$

11. a.



$p(\text{Good driver and Ticket}) = .045$

$p(\text{Good driver and No ticket}) = .855$

$p(\text{Bad driver and Ticket}) = .07$

$p(\text{Bad driver and No ticket}) = .03$

b. 5,750

c. 3,500

d. $\frac{3{,}500}{5{,}750}$

12. a. $\frac{1}{1{,}000}$

b. $\frac{729}{1{,}000}$

c. $\frac{504}{1{,}000}$

13. a. 100

b. 45

14. a. Approximately .134.

b. Approximately .804.

15. 15

16. a. .2

b. .1

c. .6

d. .2

e. No.

f. Yes.

17. a. Approximately 5%.

b. Approximately 93%.

c. About 1.

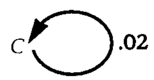
18. Mutually exclusive: rolling a number divisible by 5 and rolling a number divisible by 3, rolling a number divisible by 5 and rolling a number divisible by 2. Independent: rolling a number divisible by 2 and rolling a number divisible by 3.

Chapter 7

Lesson 7.1

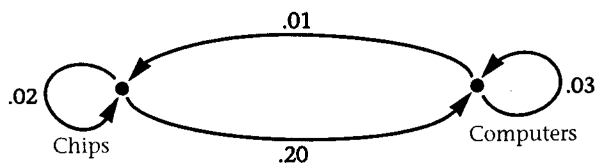
1. 25; 475; 45; 855; 2,000; 1,900; 5,000; 4,750; 2,500; 125; 7,500; 375; $0.05P$; $P - 0.05P$

2. a. 2%

b. 

c. $D = P - 0.02P$

d. $P = \$20,408$

3. a. 

b. $C =$

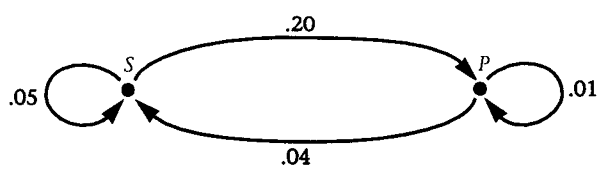
	Chips	Computers
Chips	0.02	0.20
Computers	0.01	0.03

c. \$20 chips, \$10 computers.

d. \$150 computers, \$1,000 chips.

e. \$25,510

f. \$51,546

4. a. 

b. 5 cents, 4 cents, 1 cent, 20 cents.

c. \$1 million, \$0.8 million.

d. \$0.4 million, \$8 million.

e. \$11 million, \$38.8 million.

Lesson 7.2

1. a.

$$C = \begin{array}{l} \text{Chips} \\ \text{Computers} \end{array} \overset{\begin{array}{cc} \text{Chips} & \text{Computers} \end{array}}{\begin{bmatrix} 0.02 & 0.20 \\ 0.01 & 0.03 \end{bmatrix}}$$

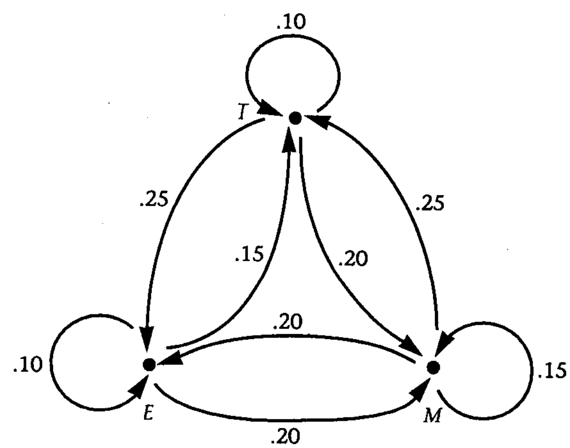
b. $$P = \begin{array}{l} \text{Chips} \\ \text{Computers} \end{array} \begin{bmatrix} \$40{,}000 \\ \$50{,}000 \end{bmatrix}$$

c. $$CP = \begin{array}{l} \text{Chips} \\ \text{Computers} \end{array} \begin{bmatrix} \$10{,}800 \\ \$1{,}900 \end{bmatrix}$$

d. $$D = \begin{array}{l} \text{Chips} \\ \text{Computers} \end{array} \begin{bmatrix} \$29{,}200 \\ \$48{,}100 \end{bmatrix}$$

e. $$P = \begin{array}{l} \text{Chips} \\ \text{Computers} \end{array} \begin{bmatrix} \$35{,}210 \\ \$72{,}528 \end{bmatrix}$$

4. a.



b.

$$C = \begin{array}{c} \text{T} \\ \text{E} \\ \text{M} \end{array} \overset{\begin{array}{ccc} \text{T} & \text{E} & \text{M} \end{array}}{\begin{bmatrix} 0.10 & 0.25 & 0.20 \\ 0.15 & 0.10 & 0.20 \\ 0.25 & 0.20 & 0.15 \end{bmatrix}}$$

Note: The entries in the matrices for parts c through f represent millions of dollars.

c.

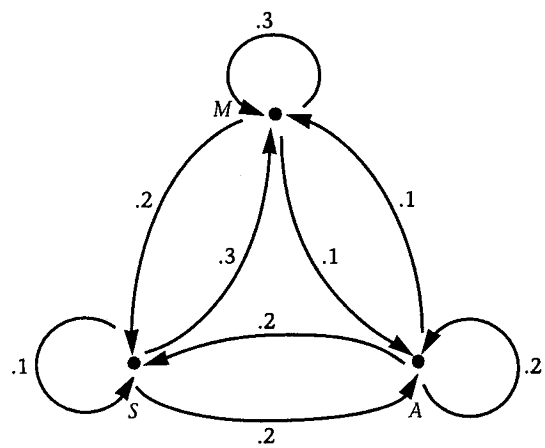
$$P = \begin{array}{c} \text{T} \\ \text{E} \\ \text{M} \end{array} \begin{bmatrix} 150 \\ 200 \\ 160 \end{bmatrix}$$

d. $CP = \begin{matrix} T \\ E \\ M \end{matrix} \begin{bmatrix} 97.0 \\ 74.5 \\ 101.5 \end{bmatrix}$

e. $D = \begin{matrix} T \\ E \\ M \end{matrix} \begin{bmatrix} 53.0 \\ 125.5 \\ 58.5 \end{bmatrix}$

f. $P = \begin{matrix} T \\ E \\ M \end{matrix} \begin{bmatrix} 218.60 \\ 195.24 \\ 239.65 \end{bmatrix}$

5. a.



b. Manufacturing is most dependent on itself and service (0.3) and least dependent on agriculture (0.1).

c. $8 million from each.

Note: The entries in the matrices for parts d and e represent millions of dollars.

d. $CP = \begin{matrix} S \\ M \\ A \end{matrix} \begin{bmatrix} 12.5 \\ 13.0 \\ 9.5 \end{bmatrix}, \quad D = \begin{matrix} S \\ M \\ A \end{matrix} \begin{bmatrix} 7.5 \\ 12.0 \\ 5.5 \end{bmatrix}$

e. $P = \begin{matrix} S \\ M \\ A \end{matrix} \begin{bmatrix} 11.03 \\ 11.61 \\ 9.21 \end{bmatrix}$

Lesson 7.3

1. a. $D_0 = [.75 \quad .25]$ $D_1 = [.625 \quad .375]$ $D_2 = [.5875 \quad .4125]$
$D_3 = [.57625 \quad .42375]$ $D_4 = [.572875 \quad .427125]$

b. $D_{10} = [.571430 \quad .428570]$
$D_{15} = [.571429 \quad .428571]$

c. Based on these results the food director can expect 57% of the students to eat in the cafeteria on any given day in the long run.

e. No.

f. $T^{15} = \begin{bmatrix} .571429 & .428571 \\ .571429 & .428571 \end{bmatrix}$

The entries in each row in T^{15} are the same as those in D_{15}.

2. a. $[.571429 \quad .428571]$

3. a. $D_0 = [1 \quad 0]$ $D_1 = [.7 \quad .3]$ $D_2 = [.61 \quad .39]$ $D_3 = [.583 \quad .417]$
$D_4 = [.5749 \quad .4251]$

b. $D_{10} = [.571431 \quad .428569]$
$D_{15} = [.571429 \quad .428571]$

After several weeks, 57% of the students will be eating in the cafeteria.

4. a. No. The sum of the entries in row 2 is greater than 1.

b. No. The matrix is not a square matrix.

c. No. Entries must be probabilities (between 0 and 1 inclusive).

d. No. The matrix is not a square matrix and the sum of the entries in row 2 is less than 1.

e. Yes.

f. No. The sum of the entries in row 2 is less than 1.

5. a. .42.

b. .08.

c. [.6 .4].

d. $\begin{bmatrix} .7 & .3 \\ .2 & .8 \end{bmatrix}$.

e. $D_7 = [.40 \quad .60]$

f. 40%.

12. a. $D_0 = [1 \quad 0 \quad 0]$

b. $D_1 = [.8 \quad .2 \quad 0]$ $D_2 = [.66 \quad .28 \quad .06]$ $D_3 = [.556 \quad .300 \quad .144]$
$D_4 = [.4748 \quad .2912 \quad .2340]$

After four days there is a 47% probability that the rat will be well, 29% probability that it will be ill, and 23% probability that it will be dead.

Lesson 7.4

1.

Best strategies	Player 1	Player 2	Strictly determined	Saddle point
a.	row 1	column 2	yes	8
b.	row 1	column 1	yes	0
c.	row 1 or 2	column 1	no	
d.	row 1	column 2	no	
e.	row 3	column 3	yes	4
f.	row 1	column 1	yes	0

2. a. Best strategies: Row 2, column 2, saddle point is 3.

b. Best strategies stay the same. Adds 4 to the saddle point.

c. Best strategies stay the same. Doubles the saddle point.

d. A conjecture: When a constant is added to each value in the payoff matrix of a strictly determined game, the best strategies stay the same and the saddle point is increased by the constant. If each value is multiplied by a constant, the best strategies stay the same and the saddle point is also multiplied by the constant.

4. a. Every other row dominates row B. Eliminate row B.
Column E dominates column G. Eliminate column G.
Row C dominates rows A and D. Eliminate rows A and D.
Column F dominates column E. Eliminate column E.
Best strategies: Row C and column F.

7.
$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 10 & -10 & -10 \\ -20 & 20 & -20 \\ -30 & -30 & 30 \end{bmatrix} \end{array}$$
No saddle point.

Lesson 7.5

1. b. The probability that both Sol and Tina will show heads is .15.

c. The probability that Tina will show tails when Sol shows heads is .35.

d. The probability that Tina will show heads when Sol shows tails is ,15.

e. The probability that both Sol and Tina will show tails is .35.

f.

Outcome	HH	HT	TH	TT
Probability	.15	.35	.15	.35
Amount won	4	−2	−3	1

g. Sol's payoff expectation for this game is −.2. This means that Sol will lose 2 pennies every 10 plays.

h. It is the same.

2. a. $A = [.75 \quad .25]$, $C = \begin{bmatrix} .3 \\ .7 \end{bmatrix}$,

$$ABC = [.75 \quad .25]\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} .3 \\ .7 \end{bmatrix} = -.2$$

b. Choose different values for the probabilities for Sol in matrix A. Calculate the expected payoff for Sol in each instance.

c. Let $A = [4 \quad .6]$ and choose different values representing probabilities for Tina in matrix C. Calculate the expected payoff for Sol in each instance.

6.

Payoff matrix: Player 1 (rows) vs. Player 2 (columns)

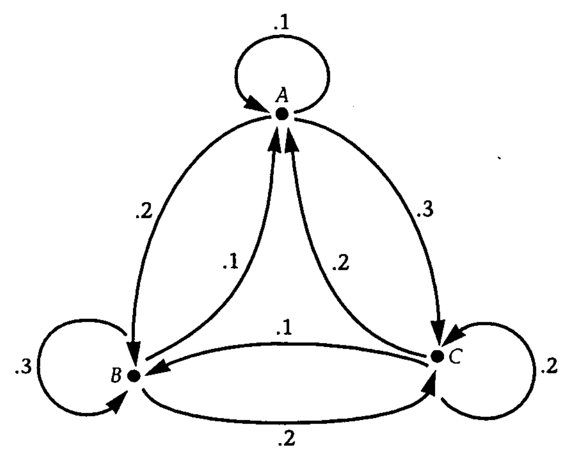
$$\begin{array}{cc} & \text{Player 2} \\ & \begin{array}{cc} 1 & 2 \end{array} \\ \text{Player 1} \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix} \end{array}$$

Best strategies: Both row and column players play 1 finger seven-twelfths of the time and 2 fingers five-twelfths of the time. The row player can expect to lose 1 cent in 12 plays or about 8 cents in 100 plays. Not a fair game since the row player will lose. In a fair game the expected payoff for both players will be the same.

8. Row 2 dominates row 1. Eliminate row 1 (making phone calls). Column 2 dominates column 1. Eliminate column 1 (making phone calls). Group against should send out mailings three-fourths of the time and go door-to-door one-fourth of the time. Group in favor should send out mailings one-half of the time and go door-to-door one-half of the time. Opposing group only gets 250 signatures (not enough to get the issue on the ballot).

Chapter 7 Review

2. a.



b.

$$CP = \begin{array}{c} A \\ B \\ C \end{array} \begin{bmatrix} 7.7 \\ 7.4 \\ 5.8 \end{bmatrix} \quad D = \begin{array}{c} A \\ B \\ C \end{array} \begin{bmatrix} 0.3 \\ 4.6 \\ 9.2 \end{bmatrix}$$

c.

$$P = \begin{bmatrix} 18.6 \\ 20.4 \\ 22.2 \end{bmatrix}$$

3. Mike's best strategy is not to bluff. The saddle point for this matrix is −2.

4. a. Mike's best strategy is to play his black card seven-twentieths of the time and his red card thirteen-twentieths of the time.
Nancy's best strategy is to play her black card nine-twentieths of the time and her red card eleven-twentieths of the time.

c.

Outcome	BB	BR	RB	RR
Probability	$\frac{63}{400}$	$\frac{77}{400}$	$\frac{117}{400}$	$\frac{143}{400}$
Amount won	7	−6	−4	3

d. The expected value of the game for Mike is $\frac{-3}{20}$.

e. Mike can expect to lose three cents every twenty hands played.

5. a. Probability of another quiz on Friday is 29%.

b. Students should expect that the teacher will start class with a quiz one-third of the time.

c. She will start class with a quickie review 40% of the time.

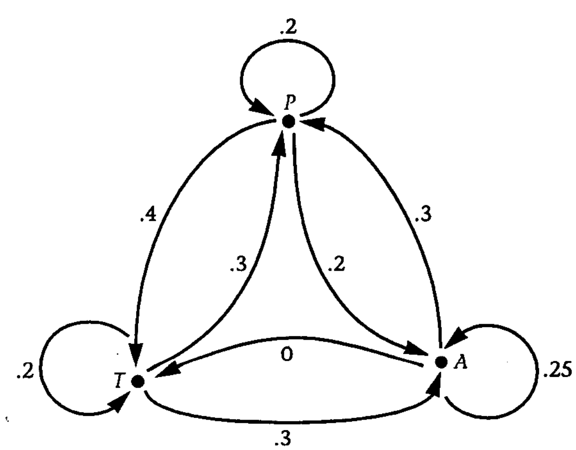
6. a.
$$C = \begin{bmatrix} .20 & .25 & .55 \\ .45 & .35 & .20 \\ .20 & .25 & .55 \end{bmatrix}$$

b. 28.5%

c. O: 27%, I: 28%, B: 45%.

d. Answers will vary. A possible answer: The clerk will know how many of each type of sandwich to order each day.

7. a.



b.

$$C = \begin{matrix} & T & P & A \\ T & 0.2 & 0.3 & 0.3 \\ P & 0.4 & 0.2 & 0.2 \\ A & 0.0 & 0.3 & 0.25 \end{matrix}$$

c. Transportation is most dependent on petroleum (0.4) and least dependent on agriculture (0.0).

d. \$1.08 million from petroleum. \$1.35 million from agriculture.

e.

$$CP = \begin{matrix} T \\ P \\ A \end{matrix} \begin{bmatrix} 16.00 \\ 16.00 \\ 11.25 \end{bmatrix} \quad D = \begin{bmatrix} 4.00 \\ 9.00 \\ 3.75 \end{bmatrix}$$ (in millions of dollars)

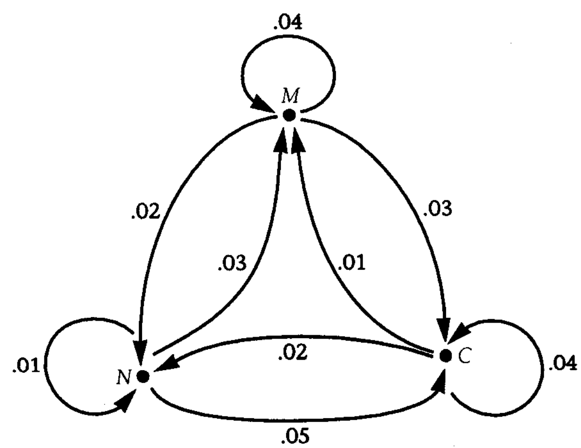
f.

$$P = \begin{bmatrix} 16.4 \\ 17.5 \\ 11.0 \end{bmatrix}$$ (in millions of dollars)

8. The best strategy for both companies is to focus on school district A.

9. The Democrats' best strategy is to go with strategy A one-fourth of the time and strategy B three-fourths of the time. The Republicans' best strategy is to go with strategy C one-half of the time and strategy D one-half of the time.
Expectation for the Democrats: 45% of undecided voters joining them.

10. a.



b.

$$P = \begin{bmatrix} \$54{,}136 \\ \$34{,}112 \\ \$42{,}941 \end{bmatrix}$$

c. $$CP = \begin{bmatrix} \$4,136 \\ \$4,112 \\ \$2,941 \end{bmatrix}$$

d. Internal consumption change: $\begin{bmatrix} \$1,005 \\ \$1,061 \\ \$804 \end{bmatrix}$ Production change: $\begin{bmatrix} \$11,005 \\ \$9,061 \\ \$12,803 \end{bmatrix}$

11. Store A: Lower prices. Store B: No change.

Chapter 8

Lesson 8.1

1. b. 1, 4, 9

c.

Number of couples	Number of handshakes	Recurrence relation
1	1	—
2	4	$H_2 = H_1 + 3$
3	9	$H_3 = H_2 + 5$
4	16	$H_4 = H_3 + 7$
5	25	$H_5 = H_4 + 9$

2. a.

Couples	Handshakes
1	0
2	2
3	6
4	12

b. $2n - 2$

c. $H_n = H_{n-1} + 2n - 2$

3. a. i. $H_n = H_{n-1} + 3$

ii. $H_n = (H_{n-1}) \cdot 2$

b. 16, 64, 36, 4320

5. a. 4

c. 0

7. a.

Term number	Number of handshakes	First differences	Second differences
1	0	—	—
2	1	1	—
3	3	2	1
4	6	3	1
5	10	4	1
6	15	5	1
7	21	6	1
8	28	7	1

10. a.

Term number	Number of bees
0	5,000
1	5,600
2	6,272
3	7,024.64
4	7,867.60

b. $B_n = 1.12B_{n-1}$

c. After 27 years, in 2014.

Lesson 8.2

2. a. $H_n = 2n^2 - 5n$

b. $H_n = 0.29 + 0.23(n - 1)$

d. $H_n = 3^{n-1}$

3. a. $T_n = T_{n-1} + n - 1$

b. 0

4. a. 1, −2, −11, −38, −119, −362

b. −2.5

6. a.

Row number	Number of seats	Total seats
1	24	24
2	26	50
3	28	78
4	30	108
5	32	140
6	34	174

b. $S_n = S_{n-1} + 2$

c. $S_n = 24 + 2n - 2$

d. 37

f. $T_n = T_{n-1} + 24 + 2n - 2$

g. $T_n = n^2 + 23n$

Lesson 8.3

1. b. i. $H_n = H_{n-1} + 3$
   iii. $H_n = (H_{n-1}) \cdot 1.2$
   iv. $H_n = H_{n-1} + H_{n-2}$

   c. i. $H_n = 2 + 3(n - 1)$
   iii. $H_n = 10(1.2^{n-1})$

5. c. $5,755.11

   g. $5,772.76

6.

Year	4.8% Monthly	5% Yearly
0	$5,000.00	$5,000.00
1	$5,245.35	$5,025.00
2	$5,502.74	$5,050.13
3	$5,772.76	$5,075.38

7. a. 74; 585

   b. 63.25; 817.5

10. \$117,463.15

11. Approximately 6.5%.

15. About \$32,500.

b. A little over \$10,100.

16. a. 1.14471

b. 2,318

Lesson 8.4

1. a. \$93,070.22

b. \$48,000

c. \$45,070.22

d. \$202,107.52, \$93,070.22

5. a.

T (in months)	T_n
0	12,000
1	11,802
2	11,602.22
3	11,400.64

b. $A_n = (A_{n-1}) \cdot 1.007 - 286$

c. It takes 50 months.

7. a. $M_n = (M_{n-1}) \cdot 1.1 - 1000$

b. At the end of the third year, \$3,345.

c. 10%

d. The fixed point is \$10,000.

8. \$10,206.75

Lesson 8.5

1. a. -3; $T_n = 4(2^{n-1}) - 3$; 2.5353×10^{30}

b. 3.5; $T_n = 1.5(3^{n-1}) + 3.5$; 2.5769×10^{47}

2. a. $B_n = (B_{n-1})\left(1 + \frac{0.08}{12}\right) + 150$

b.

Month	Balance
0	150
1	301
2	453.01
3	606.03
4	760.07

c. $-22{,}500$

d. $22{,}801\left(1 + \frac{0.08}{12}\right)^{n-1} - 22{,}500$, or $22{,}650\left(1 + \frac{0.08}{12}\right)^{n} - 22{,}500$.

e. $225,194.28

f. 472 months.

g. Approximately $333.

3. a. $B_n = (B_{n-1}) \cdot 1.01 - 220$

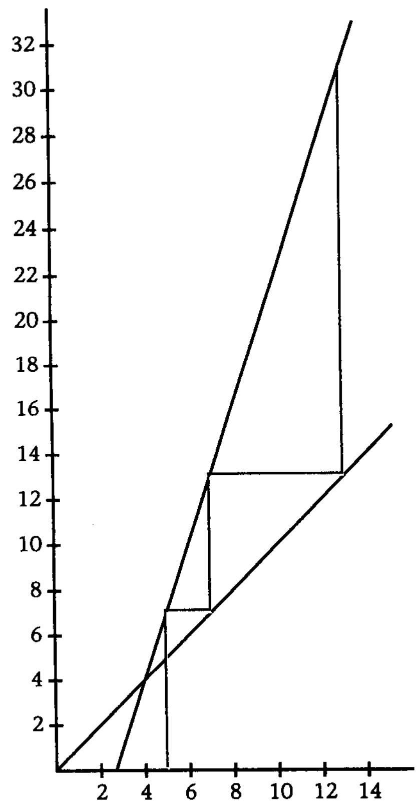
c. 22000.

e. $6763.18.

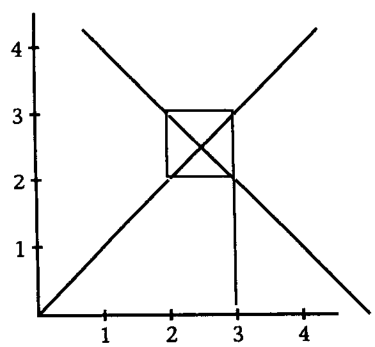
f. 61 months.

Lesson 8.6

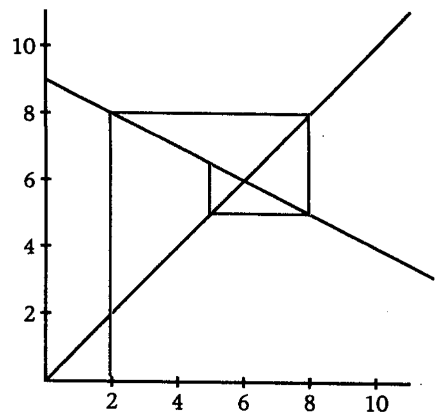
1. a.



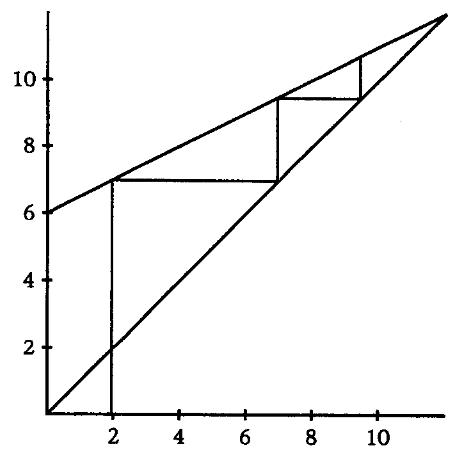
b.



c.



d.



4. a. 1

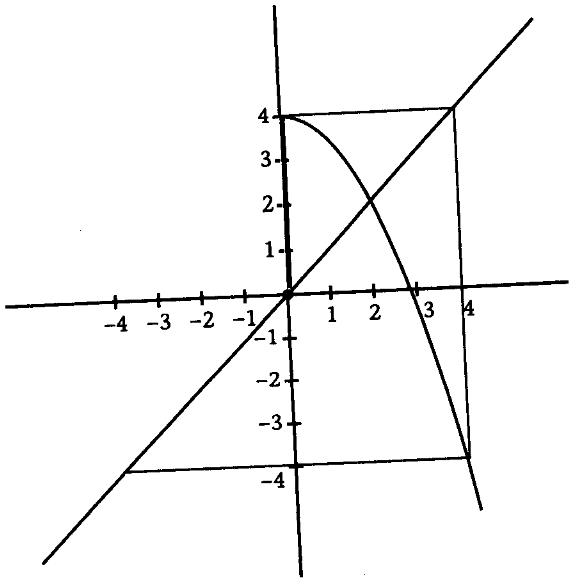
b. 3

c.

n	t_n
1	1
2	5
3	13

d. $t_n = 2t_{n-1} + 3$

6. b.



c. The behavior is unpredictable.

f. 2, −4

Chapter 8 Review

2. a. $H_n = H_{n-1} + 4$; $H_n = 2 + 4(n - 1)$; 398

b. $H_n = 3H_{n-1} - 1$; $H_n = \frac{5}{2}3^{n-1} + \frac{1}{2}$; $4.29 \cdot 10^{47}$

c. $H_n = (H_{n-1}) \cdot 2$; $H_n = 3(2^{n-1})$; $1.9014759 \cdot 10^{30}$

3. a. Arithmetic.

b. Neither.

c. Geometric.

4. a. 0; 0, 0, 0, 0

b. −0.75; −0.75, −0.75, −0.75, −0.75

c. No fixed point.

5. a. $H_n = 2(5^{n-1})$; $3.1554436 \cdot 10^{69}$

b. $H_n = 2.75(5^{n-1}) - 0.75$; $4.338735 \cdot 10^{69}$

c. $H_n = 2 + (-3)(n - 1)$; -295

6. a. Geometric.

b. Neither.

c. Arithmetic.

7. a.

N	S_n	First differences	Second differences	Third differences
1	1	—	—	—
2	5	4	—	—
3	14	9	5	—
4	30	16	7	2
5	55	25	9	2

c. $\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$; 204

8. Second degree; $H_n = n^2 - 6$

9. a.

Day	Gifts that day	Total gifts
1	1	1
2	3	4
3	6	10
4	10	20
5	15	35
6	21	56

b. $G_n = G_{n-1} + n$

$T_n = T_{n-1} + \frac{n^2}{2} + \frac{n}{2}$

c. $G_n = \frac{n^2}{2} + \frac{n}{2}$

$T_n = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$

10. a. $P_n = P_{n-1} + 0.23$

b. $P_n = 0.32 + 0.23(n - 1)$

11. a.

Month	Balance
0	\$1,000
1	\$1,004
2	\$1,008.02
3	\$1,012.05

b. $B_n = 1.004(B_{n-1})$

c. $B_n = 1000(1.004)^n$

d. 14 years, 6 months.

12. a.

Month	Balance
0	\$5,000
1	\$5,126.67
2	\$5,254.01
3	\$5,382.03

b. $B_n = (B_{n-1})\left(1 + \frac{0.064}{12}\right) + 100$

c. $B_n = 23{,}750\left(1 + \frac{0.064}{12}\right)^n - 18{,}750$

d. \$13,928.99

e. 200 months.

13. a. $B_n = 1.008(B_{n-1}) - 230$

b. $B_n = -17{,}750(1.008)^n + 28{,}750$

c. 61 months.

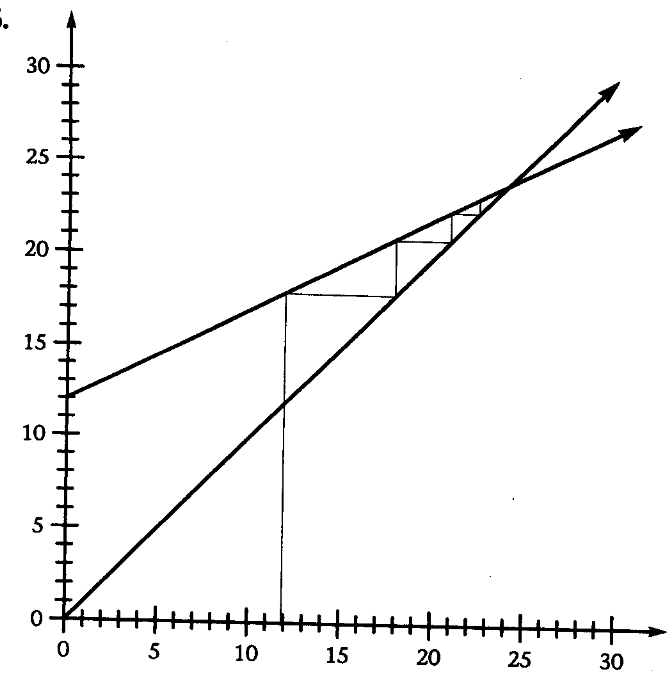
d. \$3,030

e. \$352.88

14. a. $V_n = 1.5V_{n-1} + 4{,}000$

b. $V_n = 16{,}000(1.5)^{n-1} - 8000$

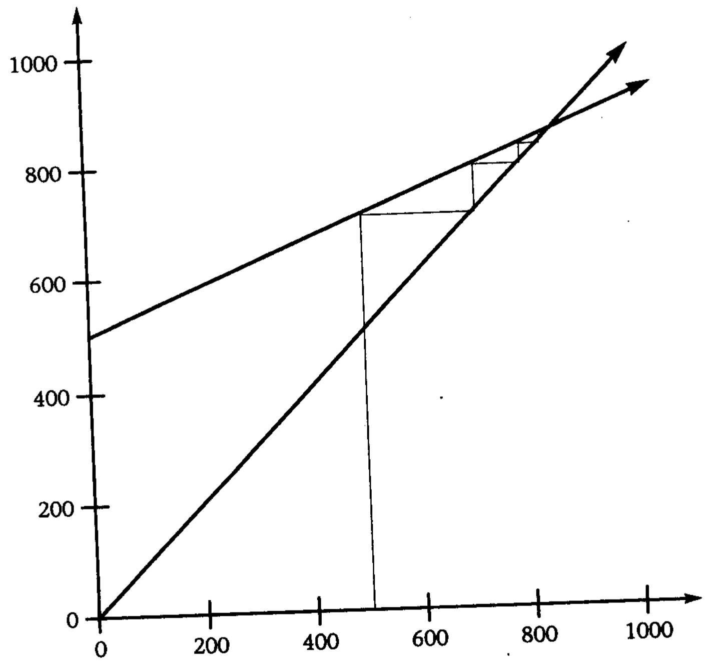
15.



16. a. $A_n = 0.4A_{n-1} + 500$

b. The amount of medication in the body stabilizes at 833 mg.

c. The cobweb would be attracted to the point (833.33, 833.33), which is the intersection of $y = x$ and $y = 0.4x + 500$, as shown in this figure.



d. The amount in the body reaches the stable value (833 mg in this case) more quickly. The stable value is probably near the optimal dosage of the drug.

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