

Analysis

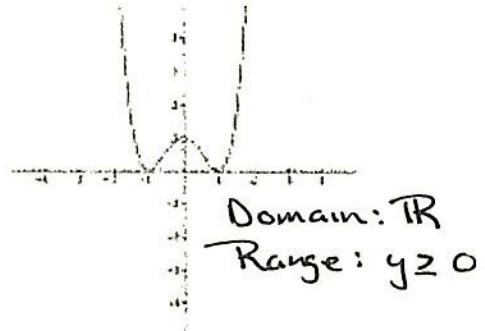
Final Review (Chapters 1,2,4,5,6)

Chapter 1: Section 1.2

1. Find the domain and range of the following:

a. $f(x) = \frac{4}{x^2 - 4}$
 Domain: $x \neq \pm 2$
 Range: $y \neq 0$

b. $f(x) = \sqrt{9 - x^2}$
 Domain: $-3 \leq x \leq 3$
 Range: $0 \leq y \leq 3$



2. Determine which of the following do represent y as a function of x:

a. $3x^2 + y = 9$

b. $y = |x| - 1$

c. $x^2 + y^2 = 16$

Section 1.3

Are the following functions odd, even, or neither?

a. $f(x) = 2x^3 - 3x^4 + 7$ ^{neither}

b. $g(x) = 3x^{4/3}$ ^{even}

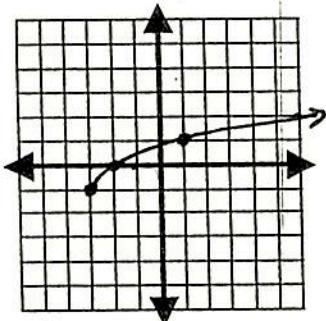
c. $h(x) = x^3 - 7x^4$ ^{odd}

Section 1.4

Use transformations of the parent function to graph each.

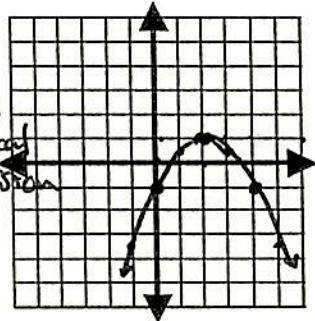
4. $g(x) = \sqrt{x+3} - 1$ using $f(x) = \sqrt{x}$.

(1) left 3
 (2) down 1



$f(x) = -\frac{1}{2}(x-2)^2 + 1$ using $g(x) = x^2$

(1) rt 2
 (2) reflect over x-axis w/ vertical compression
 (3) up 1



Tell the sequence of transformations used for each problem above.

5. What sequence of transformations will yield the graph of $h(x) = -|x - 9| + 3$ from the graph of $g(x) = |x|$?

(1) right 9

(2) reflect over x-axis

(3) up 3

Section 1.5

6. Given: $f(x) = 4 - 2x^2$ and $g(x) = 2 - x$ find the following:

a. $f(4) = 4 - 2(16)$
 $= 4 - 32$
 $= -28$

b. $(f+g)(x) = -2x^2 - x + 6$

c. $(fg)(x) = (4 - 2x^2)(2 - x)$
 $= 8 - 4x - 4x^2 + 2x^3$
 $= 2x^3 - 4x^2 - 4x + 8$

d. $g(f(-1))$
 $g(2)$
 \circ

e. $f(g(x)) = 4 - 2(2 - x)^2$
 $= 4 - 2(4 - 4x + x^2)$
 $= 4 - 8 + 8x + 2x^2$
 $= 2x^2 + 8x - 4$

f. $f(x+7) = 4 - 2(x+7)$
 $= 4 - 2(x^2 + 14x + 49)$
 $= 4 - 2x^2 - 28x - 98$
 $= -2x^2 - 28x - 94$

Section 1.6

7. Verify algebraically that the functions $f(x) = \sqrt{x^2 - 5}$ and $g(x) = x^2 + 5$ are inverses of each other.

(1) $f(g(x)) = \sqrt{(x^2 + 5)^2 - 5}$
 $= \sqrt{x^4 + 10x^2 + 25 - 5}$
 $= \sqrt{x^4 + 10x^2 + 20}$

(2) $g(f(x)) = (\sqrt{x^2 - 5})^2 + 5$
 $= x^2 - 5 + 5$
 $= x^2$

f and g are not inverses because $f(g(x)) \neq x \neq g(f(x))$

8. Given: $h(x) = \frac{1}{2}x + 5$, find $h^{-1}(x)$ if it exists.

$x = \frac{1}{2}y + 5$ $x + 5 = \frac{1}{2}y$ $2x + 10 = y$

$$h^{-1}(x) = 2x + 10$$

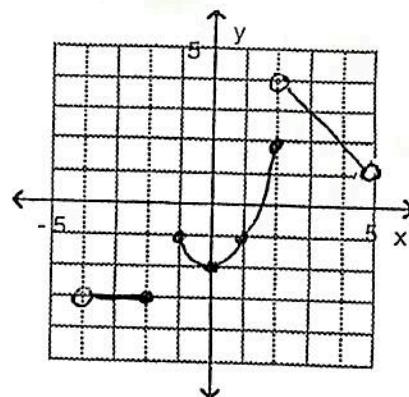
9. Given: $f(x) = \begin{cases} 2x - 1, & x \geq 1 \\ x - 5, & x < 1 \end{cases}$ find

a. $f(-2) = -7$ b. $f(1) = 1$ c. $f(5) = 9$

10. Give the domain and range for the piecewise function shown below:

Domain $[-4, -2] \cup [-1, 5)$

Range: $-3 \cup [-2, 4)$



Chapter 2

Section 2.1

1. Find the minimum point on the graph of $f(x) = 2x^2 - 4x + 14$ by writing $f(x)$ in the form $y = a(x - h)^2 + k$

$$f(x) = 2(x - 1)^2 + 12$$

2. Find the equation of a parabola with the given vertex and through the given point

$$= a(x-h)^2 + k \quad \text{a. Vertex } (-2, 5), \text{ point } (0, 9)$$

$$9 = a(0+2)^2 + 5$$

$$4 = 4a$$

$$1 = a$$

$$\boxed{y = (x+2)^2 + 5}$$

$$\quad \text{b. Vertex } (-4, -1), \text{ point } (-2, 4)$$

$$4 = a(-2+4)^2 - 1$$

$$5 = 4a$$

$$\frac{5}{4} = a$$

$$\boxed{y = \frac{5}{4}(x+4)^2 - 1}$$

3. Describe the left and right behavior of the graph $f(x) = -x^5 + 2x^2 - 1$ using mathematical symbolism.

$$f(x) \rightarrow -\infty \text{ as } x \rightarrow \infty$$

$$f(x) \rightarrow \infty \text{ as } x \rightarrow -\infty$$

Section 2.2 and 2.3

4. Given $f(x) = 2x - 1$ and $g(x) = 6x^3 + 7x^2 - 15x + 6$

$$\text{a. Divide } g(x) \text{ by } f(x) = 3x^2 + 5x - 5 + \frac{1}{2x-1}$$

$$\begin{array}{r} 3x^2 + 5x - 5 \\ 2x-1 \overline{)6x^3 + 7x^2 - 15x + 6} \\ - 6x^3 + 3x^2 \\ \hline 10x^2 - 15x + 6 \\ - 10x^2 - 5x \\ \hline - 16x + 6 \\ + 16x + 5 \\ \hline 1 \end{array}$$

b. Is $f(x)$ a factor of $g(x)$ = no the remainder is not zero.

5. According to the Rational Root Theorem which of the following values cannot be a root of the polynomial

$$f(x) = 2x^3 - 7x^2 + 5x - 6? \quad \frac{1}{2}, \left(\frac{2}{3}\right), -3, 6.$$

6. Find a 3rd degree polynomial with zeros: 1, 0, -3

$$P(x) = x(x-1)(x+3)$$

$$P(x) = x(x^2 + 2x - 3)$$

$$P(x) = x^3 + 2x^2 - 3x$$

7. Find a 4th degree polynomial with real coefficients that has zeros: -2, 1, $3i$, $-3i$

$$P(x) = (x+2)(x-1)(x-3i)(x+3i)$$

$$P(x) = (x^2 + x - 2)(x^2 + 9)$$

$$P(x) = x^4 + x^3 - 2x^2 + 9x^2 + 9x - 18$$

$$P(x) = x^4 + x^3 + 7x^2 + 9x - 18$$

8.. Show your work using synthetic division and factoring to find all the real roots.

a. $f(x) = x^3 - 12x^2 + 40x - 24$ all positive

	1	-12	40	-24
1	1	-11	29	5
2	1	-10	20	16
3	1	-9	13	15
4	1	-8	8	8
6	1	-6	4	0

$$x^2 - 6x + 4 = 0$$

$$x^2 - 6x + 9 = -4 + 9$$

$$(x-3)^2 = 5$$

$$x-3 = \pm \sqrt{5}$$

$$\boxed{x=6} \quad \boxed{x=3 \pm \sqrt{5}}$$

b. $g(x) = x^4 + 3x^3 - 9x^2 + 3x - 10$

	1	3	-9	3	-10
1	1	4	-5	-2	-12
2	1	5	1	5	6

$$x^3 + 5x^2 + x + 5 = 0$$

$$x^2(x+5) + 1(x+5) = 0$$

$$(x+5)(x^2+1) = 0$$

$$x^2 + 1 = 0$$

$$x^2 = -1$$

$$\boxed{x=2} \quad \boxed{x=-5} \quad \boxed{x=\pm i}$$

Section 2.4

9. Simplify the following:

a. $\frac{(2+3i)(1+i)}{(1-i)(1+i)}$

$$\frac{2+3i+3i+3i^2}{1+1}$$

$$\boxed{-1+5i}$$

b. $6i^{17} + 4i^{20}$

$$6(i^4)^4 i^1 + 4(i^2)^{10}$$

$$6(i^4)^8 i + 4(-1)^{10}$$

$$\frac{6i+4}{4+6i}$$

c. i^{207}

$$i^{204} i$$

$$(i^2)^{103} i$$

$$\boxed{-1-i}$$

Section 2.5

10. Factor each as linear factors irreducible over the following number sets:

a. $f(x) = 4x^4 - 19x^2 - 5$

b. $f(x) = x^4 + 7x^2 - 18$

Rationals: $(4x^2 + 1)(x^2 - 5)$

$(x^2 - 2)(x^2 + 9)$

Reals: $(4x^2 + 1)(x - \sqrt{5})(x + \sqrt{5})$

$(x - \sqrt{2})(x + \sqrt{2})(x^2 + 9)$

Complex: $(2x+i)(2x-i)(x-\sqrt{5})(x+\sqrt{5})$

$(x - \sqrt{2})(x + \sqrt{2})(x - 3i)(x + 3i)$

Section 2.6 and 2.7

11. Find the following without using the graphing functions on your calculator:

$$f(x) = \frac{x}{x^2 - x - 2} = \frac{x}{(x-2)(x+1)}$$

y-intercept:

$$(0, 0)$$

x-intercepts:

$$(0, 0)$$

holes:

none

vertical asymptotes:

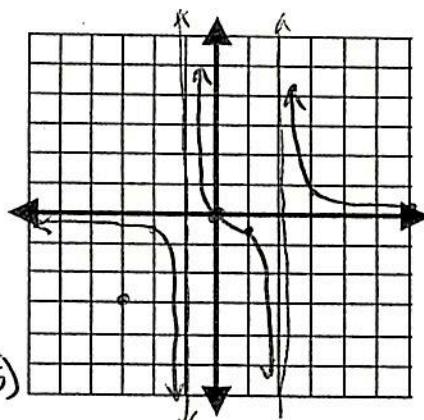
$$x = 2 \quad x = -1$$

non-vertical asymptote:

$$y = 0$$

Plot at least 3 additional points.

$$(-2, -5) \quad (1, -5) \quad (3, .75)$$



$$f(x) = \frac{x^2 - 6x + 8}{x^2 - 4} = \frac{(x-4)(x-2)}{(x+2)(x-2)}$$

$$r(x) = \frac{x-4}{x+2}$$

y-intercept:

$$(0, -2)$$

x-intercepts:

$$(4, 0)$$

holes:

$$(2, -\frac{1}{2})$$

vertical asymptotes:

$$x = -2$$

non-vertical asymptote:

$$y = 1$$

Plot at least 3 additional points.

$$(-1, -5) \quad (-3, 7)$$

$$(-4, 6)$$

$$(-5, 3)$$

$$f(x) = \frac{3x^2 + 1}{x}$$

y-intercept:

none

x-intercepts:

none

holes:

none

vertical asymptotes:

$$x = 0$$

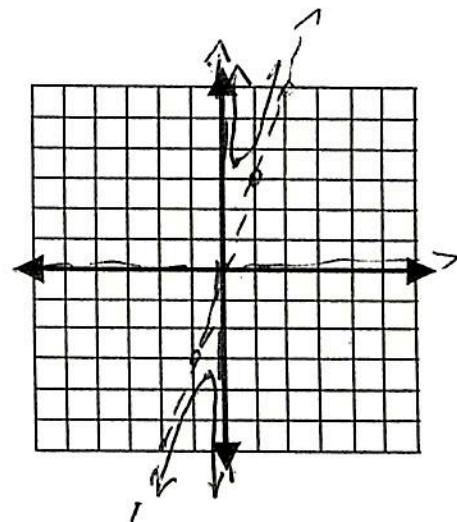
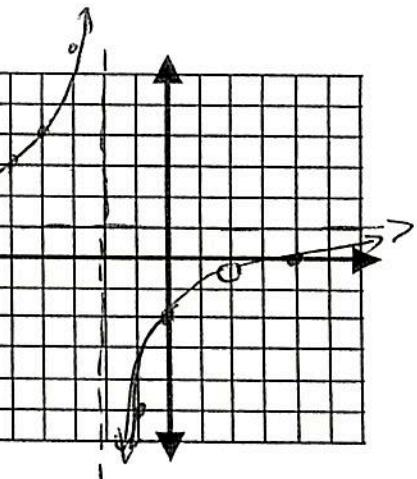
non-vertical asymptote:

$$y = 3x$$

Plot at least 3 additional points.

$$(-1, -4) \quad (1, 4)$$

$$(-.5, -3.5) \quad (.5, 3.5)$$



Chapter 4:

Show all work in a clear and organized manner.

1. Determine two coterminal angles and the reference angle for 980° .

$$260^\circ, -100^\circ \quad \boxed{\text{ref: } 80^\circ}$$

2. Determine two coterminal angles and the reference angle for $-\frac{13}{3}\pi + \frac{6}{3}\pi$

$$-\frac{7}{3}\pi, -\frac{1}{3}\pi, \frac{5}{3}\pi \quad \boxed{\text{ref: } \frac{1}{3}\pi}$$

3. Given a circle with radius 9 inches and central angle 120° , find the exact length of the intercepted arc.

$$\begin{aligned} S &= \theta R \\ S &= \frac{2\pi}{3}(9) = \boxed{S = 6\pi} \end{aligned} \quad \downarrow \quad \frac{2}{3}\pi$$

4. Find the radian measure of the central angle of a circle with radius 6 feet that intercepts an arc of 15 feet.

$$\begin{aligned} S &= \theta R \\ 15 &= 6\theta \\ \frac{15}{6} &= \theta \end{aligned} \quad \boxed{\theta = 2.5 \text{ radians}}$$

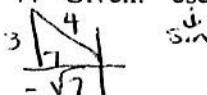
5. A CD with a diameter of 6 inches makes 74 revolutions per second. Find the linear speed of the CD (in miles per hour) rounded to the nearest hundredth. (1 mile = 5280 feet) $C = 2\pi r = d\pi = 6\pi \text{ in}$

$$\begin{array}{c|c|c|c|c|c|c} 74 \text{ rev} & 6\pi \text{ in} & 1 \text{ ft} & 1 \text{ mile} & 60 \text{ sec} & 60 \text{ min} & \approx 25.23\pi \text{ miles} \\ \hline \text{sec} & 1 \text{ rev} & 12 \text{ in} & 5280 \text{ ft} & 1 \text{ min} & 1 \text{ hr} & \hline \end{array} \quad \boxed{\text{miles} \approx 79.25 \frac{\text{miles}}{\text{hr}}}$$

6. An eighteen inch in diameter saw blade rotates at 1200 revolutions per minute. Find the angular speed of the blade in radians per second. Round to the nearest hundredth.

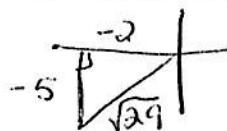
$$\begin{array}{c|c|c} 1200 \text{ rev} & 2\pi \text{ rad} & 1 \text{ min} \\ \hline \text{min} & 1 \text{ rev} & 60 \text{ sec} \end{array} = 40\pi \frac{\text{rad}}{\text{sec}} \approx 125.66 \frac{\text{rad}}{\text{sec}}$$

7. Given: $\csc \theta = \frac{4}{3}$ and $\tan \theta < 0$ find:



$$\sin \theta = \frac{3}{4} \quad \cos(\theta) = -\frac{\sqrt{7}}{4} \quad \csc(-\theta) = -\frac{4}{3}$$

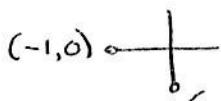
8. Given: $\cot \theta = \frac{2}{5}$ and $\cos \theta < 0$ find:



$$\sin \theta = -\frac{5\sqrt{29}}{29} \quad \tan\left(\frac{\pi}{2} - \theta\right) = \frac{2}{5} \quad \cot(-\theta) = -\frac{2}{5}$$

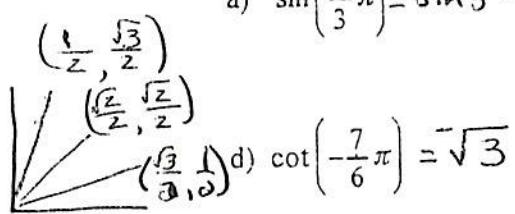
9. Which quadrant will θ be in when: $\sin \theta < 0$ and $\tan \theta < 0$? $\boxed{4}$

10. Which quadrant will θ be in when: $\cos \theta < 0$ and $\sin \theta > 0$? $\boxed{2}$



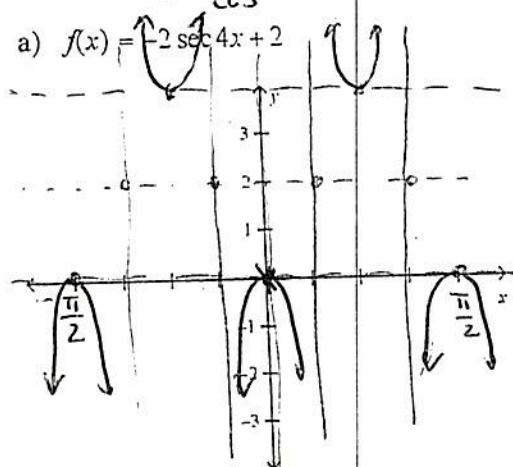
11. Use your unit circle to evaluate each:

a) $\sin\left(\frac{7}{3}\pi\right) = \sin\frac{1}{3}\pi = \frac{\sqrt{3}}{2}$ b) $\cos\left(\frac{3}{2}\pi\right) = 0$ c) $\tan\left(-\frac{\pi}{4}\right) = -1$

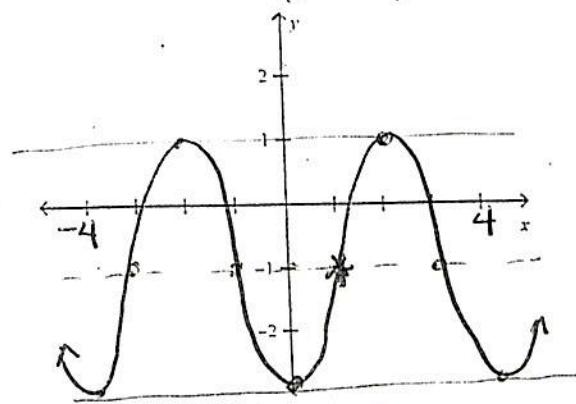


d) $\cot\left(-\frac{7}{6}\pi\right) = -\sqrt{3}$ e) $\sec(225^\circ) = -\sqrt{2}$ f) $\csc(180^\circ) = \text{und.}$

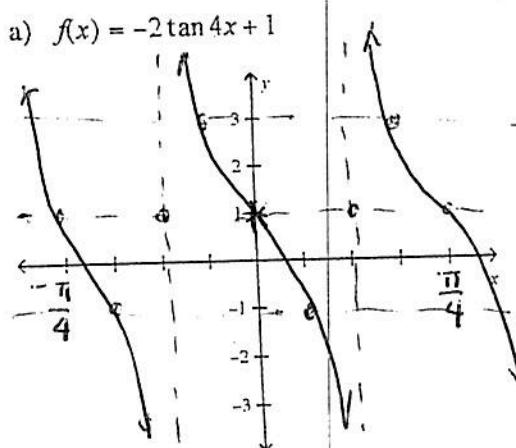
12. List critical points or clearly label axes as you graph two full periods of:



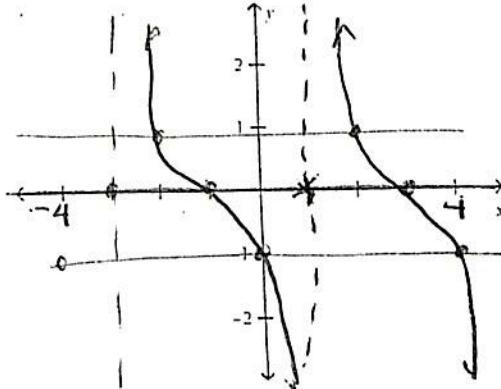
b) $f(x) = 2 \sin\left(\frac{\pi}{2}x - \frac{\pi}{2}\right) - 1 = 2 \sin\frac{\pi}{2}(x-1) - 1$



13. List critical points or clearly label axes as you graph two full periods of:



b) $f(x) = \cot\left(\frac{\pi}{4}x - \frac{\pi}{4}\right) = \cot\frac{\pi}{4}(x-1)$



$P = \pi \div \frac{\pi}{4}$

$P = \pi \approx \frac{4}{\pi}$

$P = 4$

14. For each trig function state the period and the range.

a) $f(x) = 2 \sin\left(\frac{\pi}{2}x - \frac{\pi}{2}\right) + 3$

period = 4

Range [1, 5]

b) $f(x) = -3 \cos 4(x - 1) - 5$
period = $\frac{\pi}{2}$

Range [-8, -2]

c) $f(x) = \tan 3x$
period = $\frac{\pi}{3}$

Range = \mathbb{R}

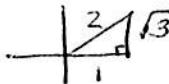
15. Evaluate each.

a) $\arcsin\left(\sin \frac{2}{3}\pi\right) = \frac{\pi}{3}$

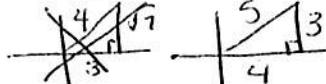
b) $\arccos\left(\cos \frac{5}{6}\pi\right) = \frac{5}{6}\pi$

c) $\arctan\left(\tan \frac{5}{3}\pi\right) = -\frac{\pi}{3}$

d) $\tan\left(\arcsin \frac{\sqrt{3}}{2}\right) = \sqrt{3}$



e) $\cos\left(\arctan \frac{3}{4}\right) = \frac{4}{5}$



f) $\arcsin(\tan 45^\circ)$

$\arcsin(1) = 90^\circ = \frac{\pi}{2}$

Chapter 5:

16. Simplify:

a) $\sec x \cos\left(\frac{\pi}{2} - x\right)$

$\cancel{\sec x} \cdot \sin x$
 $\frac{1}{\cos x} \cdot \sin x$

$\boxed{\tan x}$

c) $\cos\left(\frac{\pi}{2} - x\right) \sec x + \frac{1}{\cos x \sec\left(\frac{\pi}{2} - x\right)}$

$\sin x \cdot \frac{1}{\cos x} + \frac{1}{\cos x \cdot \csc x}$
 $\tan x + \frac{\sin x}{\cos x}$

$\boxed{2\tan x}$

e) $\frac{\tan^2 \theta}{\sin \theta \sec \theta}$

$\frac{\tan^2 \theta}{\sin \theta / \cos \theta}$

$\frac{\tan^2 \theta}{\tan \theta}$

$\boxed{\tan \theta}$

b) $\frac{1 + \cos x}{\sin x} + \frac{\sin x}{1 + \cos x} (1 - \cos x)$

$\frac{1 + \cos x}{\sin x} + \frac{\sin x (1 - \cos x)}{\sin^2 x}$

$\frac{1 + \cos x + 1 - \cos x}{\sin x}$

$\frac{2}{\sin x} = \boxed{2 \csc x}$

d) $\cos x - \cos x \sin^2 x$

$\cos x (1 - \sin^2 x)$

$\cos x \cdot \cos^2 x$

$\cos^3 x$

f) $\sin^4 \theta - \cos^4 \theta$

$(\sin^2 \theta - \cos^2 \theta)(\sin^2 \theta + \cos^2 \theta)$

$(\sin^2 \theta - \cos^2 \theta) \quad 1$

$\sin^2 \theta - \cos^2 \theta$

17. Verify.

$$a) \frac{\sec^2 \theta}{\cot \theta} - \tan^3 \theta = \tan \theta$$

$$\tan \theta (1 + \tan^2 \theta) - \tan^3 \theta$$

$$\tan \theta + \tan^3 \theta - \tan^3 \theta$$

$$\tan \theta$$

✓

$$c) \sec \theta + \tan(-\theta) \sin \theta = \cos \theta$$

$$\sec \theta - \tan \theta \sin \theta$$

$$\begin{aligned} \frac{1}{\cos \theta} &= \frac{\sec \theta}{\cos \theta}, \sin \theta \\ 1 &= \frac{\sin^2 \theta}{\cos \theta} \\ \frac{\cos \theta}{\cos \theta} &= \frac{\cos \theta}{\cos \theta} \\ \cos \theta &\checkmark \end{aligned}$$

18. Find all solutions of the following equations in the interval $[0, 2\pi)$:

$$a) 4\cos^2 x - 3 = 0$$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$

$$\boxed{x = \frac{\pi}{6}, \frac{5}{6}\pi, \frac{7}{6}\pi, \frac{11}{6}\pi}$$

$$c) \tan 3x = \sqrt{3}$$

$$3x = \frac{\pi}{3} + n\pi$$

$$x = \frac{\pi}{9} + \frac{n\pi}{3}$$

$$x = \frac{\pi}{9} + \frac{3}{7}\pi n$$

$$x = \frac{\pi}{9}, \frac{4}{9}\pi, \frac{7}{9}\pi$$

$$\frac{10}{9}\pi, \frac{13}{9}\pi, \frac{16}{9}\pi$$

19. Find all (general) solutions for the following equations:

$$a) \cos 2x - \cos x = 0$$

$$2\cos^2 x - 1 - \cos x = 0$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$(2\cos x + 1)(\cos x - 1) = 0$$

$$\cos x = -\frac{1}{2} \quad \cos x = 1$$

$$x = \frac{\pi}{3} + 2\pi n$$

$$x = \frac{5}{3}\pi + 2\pi n$$

$$x = 0 + 2\pi n$$

$$b) \frac{1 + \tan \theta}{\sin \theta} - \sec \theta = \csc \theta$$

$$\frac{1}{\sin \theta} + \tan \theta, \frac{1}{\sin \theta} - \sec \theta$$

$$\csc \theta + \frac{\sin \theta}{\cos \theta}, \frac{1}{\sin \theta} - \frac{1}{\cos \theta}$$

$$\csc \theta + \frac{1}{\cos \theta} - \frac{1}{\cos \theta}$$

csc

✓

$$d) \frac{\tan x + \cot y}{\tan x \cot y} = \tan y + \cot x$$

$$\frac{1}{\cot y} + \frac{1}{\tan x}$$

$$\tan y + \cot x$$

✓

$$\sec^2 x - \sec x - 2 = 0$$

$$(\sec x + 1)(\sec x - 2) = 0$$

$$\sec x = -1 \quad \sec x = 2$$

$$\cos x = -1 \quad \cos x = \frac{1}{2}$$

$$x = \pi, \frac{\pi}{3}, \frac{5}{3}\pi$$

$$d) \sin 2x = \sin x$$

$$2\sin x \cos x - \sin x = 0$$

$$\sin x(2\cos x - 1) = 0$$

$$\sin x = 0 \quad \cos x = \frac{1}{2}$$

$$x = 0, \pi, \frac{\pi}{3}, \frac{5}{3}\pi$$

$$b) 2\cos 2x - \sqrt{2} = 0$$

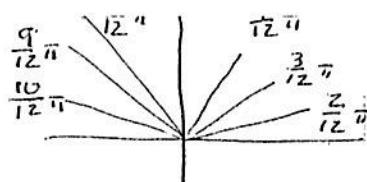
$$\cos 2x = \frac{\sqrt{2}}{2}$$

$$2x = \frac{\pi}{4} + 2\pi n \quad 2x = \frac{7}{4}\pi + 2\pi n$$

$$\boxed{x = \frac{\pi}{8} + \pi n \quad x = \frac{7}{8}\pi + \pi n}$$

$$\tan \frac{9}{12}\pi = -1$$

$$\tan \frac{2}{12}\pi = \frac{1}{\sqrt{3}}$$



20. Find the exact value of each:

a) $\sin 105^\circ$

$$\sin(60 + 45)$$

$$\sin 60 \cos 45 + \cos 60 \sin 45 \approx 45$$

$$\frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2} + \frac{1}{2}, \frac{\sqrt{2}}{2}$$

$$\boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$$

b) $\tan \frac{11}{12}\pi$

c) $\cos 285^\circ = \cos(225 + 60)$

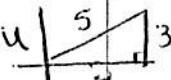
$$\cos 225 \cos 60 - \sin 225 \sin 60$$

$$-\frac{\sqrt{2}}{2} \frac{1}{2} - \frac{-\sqrt{2}}{2} \frac{\sqrt{3}}{2}$$

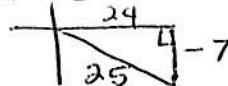
$$\boxed{-\frac{\sqrt{2} + \sqrt{6}}{4}}$$

21. Given: $\tan(u) = \frac{3}{4}$, $0 < u < \frac{\pi}{2}$

Find: $\sin(u + v)$



and $\sec(v) = \frac{25}{24}$, $\frac{3}{2}\pi < v < 2\pi$



$\sin u \cos v + \cos u \sin v$

$$\frac{3}{5} \left(\frac{24}{25}\right) + \frac{4}{5} \left(-\frac{1}{25}\right)$$

$$\frac{72 - 25}{125}$$

$$\boxed{\frac{44}{125}}$$

22. Given: $\cos u = \frac{3}{5}$; find $\cos(2u)$.

$$2\cos^2 u - 1$$

$$2\left(\frac{3}{5}\right)^2 - 1$$

$$2\left(\frac{9}{25}\right) - \frac{25}{25}$$

$$\frac{18 - 25}{25}$$

$$\boxed{\frac{-7}{25}}$$